

Theoretic and Experimental Comparison of Root-Mean-Square and Mean-Absolute-Value Electromyogram Amplitude Detectors

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Abstract — It is typically assumed that the probability density of the surface electromyogram (EMG) is Gaussian. This assumption leads to root-mean-square (RMS) processing as the maximum likelihood estimator of the EMG amplitude. Contrary to this theoretical formulation, recent experimental work has found mean-absolute-value (MAV) processing to be superior to RMS. This paper reviews RMS processing with the Gaussian model and then derives the expected (inferior) performance of MAV processing with the Gaussian model. Next, a new model for the surface EMG waveform, using a Laplacian density, is presented. It is shown that the MAV processor is the maximum likelihood estimator of the EMG amplitude for the Laplacian model. Lastly, experimental data from isometric, constant-force, non-fatiguing contractions were examined. On average, the Laplacian density best fit the experimental data (although results varied with subject). For amplitude estimation, MAV processing was clearly superior to RMS processing.

I. Introduction

Recent comparisons of root-mean-square (RMS) processing to mean-absolute-value (MAV) processing have found MAV processing to be equal to or better than RMS processing [1]. This result is contrary to the Gaussian model of the electromyogram (EMG). In this report, some theoretic considerations related to the MAV processor are presented and then experimental data are examined with respect to these considerations. First, the performance of the MAV processor is determined using the Gaussian model. Performance inferior to RMS processing is predicted. Second, a new model for the surface EMG, using a Laplacian density, is presented. This model is notable, since it is shown that optimal estimation of the EMG amplitude based on the Laplacian model is accomplished with the MAV processor. Finally, experimental data from isometric, constant-force, non-fatiguing contractions are used to compare the two surface EMG models.

II. Models of the Surface Electromyogram

A. Standard Results of the Gaussian Model

The standard model assumes that the surface EMG sampled during isometric, constant-force, non-fatiguing contractions is a zero-mean, wide sense stationary, band-limited, correlation-ergodic, random process. The probability density of the samples is assumed jointly

Gaussian distributed [2]–[5]. For the purposes of this report the EMG bandwidth affects only the number of degrees of freedom in the data, N . Thus, without loss of generality, all further discussion will consider only the case of uncorrelated EMG samples. With this model, [2] has shown that the maximum likelihood (ML) estimate of the EMG amplitude is the RMS processor, denoted herein as \hat{s}_{RMS} . Note that \hat{s}_{RMS} is distributed as a Chi random variable [6; pp. 96, 187]. The signal-to-noise-ratio (SNR) of this estimator, defined as the mean value of an estimate divided by its standard deviation, is closely approximated as [3]

$$SNR_{\hat{s}_{RMS}} \cong \sqrt{2N}$$

The SNR is a standard metric used to compare processor performance.

B. MAV Processor Performance, Gaussian Model

The SNR performance of the MAV processor, denoted $SNR_{\hat{s}_{MAV}}$ can also be derived from the white Gaussian model of EMG. The complete derivation is available in [1], where it is shown that

$$SNR_{\hat{s}_{MAV}} = \sqrt{\left(\frac{2}{\pi-2}\right)N}$$

C. Laplacian Model

Alternatively, the MAV processor can be shown to be optimal if the underlying probability density function (PDF) for the random process is distributed as a Laplace random variable, and successive samples of the EMG are independent. (Note that independent is a stronger assumption than uncorrelated.) A zero mean Laplace random variable x has the PDF

$$p_x(X) = \frac{\sqrt{2}}{2\sigma_x} e^{-\frac{\sqrt{2}}{\sigma_x}|x|} \quad -\infty \leq X \leq \infty, \sigma_x > 0$$

where σ_x is the standard deviation. In the discrete-time case, let N independent EMG waveform random variables be denoted $m_1, m_2, m_3, \dots, m_N$. Given that the EMG amplitude has the known value $s_{Laplace}$, the conditional joint PDF for the N random variables (arranged as the random vector \underline{m}) is the product of the individual conditional PDFs;

$$P_{\underline{m}|s_{Laplace}}(\underline{M}|\hat{s}_{Laplace}) = \left[\frac{\sqrt{2}}{2\hat{s}_{Laplace}}\right]^N e^{-\frac{\sqrt{2}}{\hat{s}_{Laplace}}\sum_{i=1}^N |M_i|} \quad -\infty \leq M_i \leq \infty$$

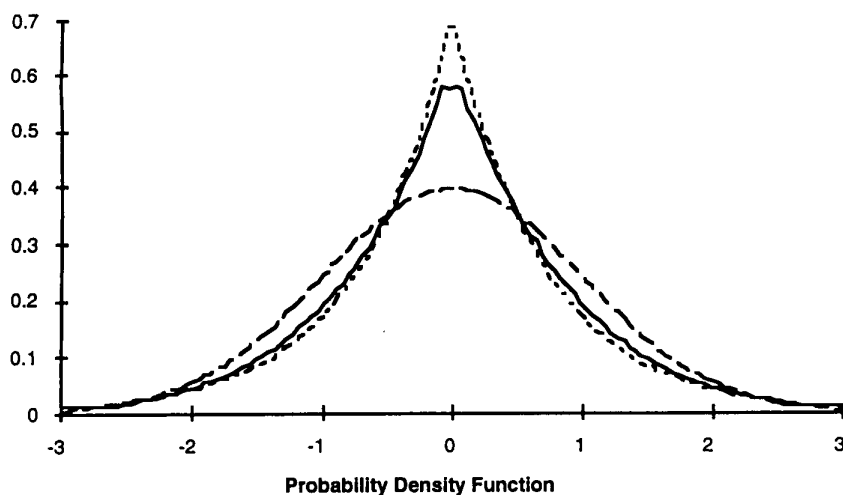


Figure 1: Probability density functions. Experimental density (solid line) is the average of 660 recordings. Dotted line indicates the Laplacian density, dashed line indicates the Gaussian density.

where \underline{M} is the vector of EMG samples and $\hat{s}_{Laplace}$ is the amplitude estimate. The ML estimate of the standard deviation is the value of $\hat{s}_{Laplace}$ which maximizes the above density. Since a monotonic transformation of the density does not alter the location of the maximum, the natural logarithm of the density can be taken. Differentiating the natural logarithm of the density with respect to $\hat{s}_{Laplace}$, setting this derivative to zero and solving for $\hat{s}_{Laplace}$ gives the desired estimate;

$$\hat{s}_{Laplace} = \frac{\sqrt{2}}{N} \sum_{i=1}^N |M_i|$$

This is the MAV processor with a constant scaling factor. (A scaling factor does not alter the SNR performance of an estimator.)

To determine the SNR performance of this estimator, consider the probability distribution which results from the processing specified by $\hat{s}_{Laplace}$. First, taking the magnitude of a zero-mean Laplace random variable (RV) of standard deviation σ_x gives an exponential RV with mean value $\sigma_x/2$. Second, the sum of N such independent, identically distributed exponential RVs gives an N degree of freedom Erlang RV with mean value $N\sigma_x/\sqrt{2}$ and variance $N\sigma_x^2/2$ [7; pp. 106–107, 273]. Finally, scaling by $\sqrt{2}/N$ results in $\hat{s}_{Laplace}$ being distributed as an N degree of freedom Erlang RV with mean value σ_x and variance σ_x^2/N [6; p. 96]. Thus, the SNR performance for this processor is

$$SNR_{\hat{s}_{Laplace}} = \sqrt{N}$$

III. Experimental Study

A. Apparatus and Methods

The experimental apparatus and methods have been described in detail elsewhere [1], [4], [5]. Briefly, a subject was seated and secured into a straight-back chair. The subject's right wrist was cuffed to a cantilever beam (which was rigidly attached to the chair). Torque about the elbow was measured by a complete strain gauge Wheatstone bridge affixed on the beam. Commercial electrode-amplifiers were placed side by side transversely across the biceps or triceps muscles of the elbow, the two electrode contacts of each electrode-amplifier being oriented along the direction of action potential conduction. The electrode-amplifiers were located midway between the elbow and the midpoint of the upper arm, clustered about the muscle midline. Each electrode-amplifier consisted of a pair of 4 mm diameter, stainless steel, hemi-spherical electrode contacts separated by a distance of 15mm (center to center). The distance between adjacent electrode-amplifiers was 1.75cm.

Five subjects (four male/one female, aged 23–37 years) each participated in one experiment. Informed consent was received from each subject. A sequence of five sets of constant-force, isometric contractions was conducted. Each set consisted of four randomized trials, one trial each at 10, 25, 50 and 75% maximum voluntary contraction (MVC), with a rest period of two minutes between trials. Up to eight channels of EMG data were each sampled at 2048 Hz for five seconds per trial, using a 12-bit A/D converter, and processed off-line. A total of 660 single site recordings were available for analysis. Previous experimental studies based on the data from these trials are described in [4], [5].

Subject	Muscle	Total Number of Records	Mean ± Std. Dev. Area Difference Between Experimental Density and:		Mean ± Std. Dev. SNR Using Processor:	
			Laplace	Gauss	MAV	RMS
AD	Triceps	100	.226±.119	.293±.127	12.1±4.0	11.3±4.9
CC	Triceps	120	.224±.126	.313±.125	12.1±3.4	11.3±3.8
DA	Biceps	160	.258±.049	.182±.067	12.1±2.4	11.8±2.5
EA	Biceps	120	.259±.047	.176±.083	10.9±2.9	10.7±2.8
FA	Triceps	160	.230±.094	.439±.097	10.0±2.0	8.8±1.7
Total		660	.240±.091	.284±.143	11.4±3.0	10.7±3.3

Table I: Density Area Differences and SNRs Tabulated by Subject

B. Methods of Analysis

For each of the 660 recordings, a sample histogram (density estimate) was constructed. The 10240 EMG waveform samples from each record were normalized (sample mean adjusted to zero, sample variance adjusted to one) and a histogram of the normalized samples formed. The histogram used 500 bins, equally spaced over the normalized range of -12 to +12. The absolute area difference between the histogram versus a Gaussian density (and also versus a Laplacian density) was computed using the 500 histogram values. Lastly, all 660 histograms were averaged to provide a composite density estimate.

In addition to density estimates, RMS and MAV processing was performed on each of the records. A fixed window length of 500 samples (245 ms) was used. SNRs of each EMG amplitude were computed as the square root of the ratio of the squared amplitude estimate sample mean divided by the amplitude estimate sample variance. To avoid a startup artifact, the first 500 samples of each amplitude estimate were discarded prior to computing the SNR.

C. Results

Fig. 1 shows the composite EMG density estimate as well as the analytic densities for Gaussian and Laplacian distributions. The area difference between the composite experimental density and the Gaussian density was 0.206, while this difference for the Laplacian density was 0.078 (2.6 times smaller). Table I tabulates, *by subject*, the density area differences (experimental vs. Gauss; experimental vs. Laplace) and the EMG amplitude estimate SNRs. Table II tabulates the number of times each density (Gauss/Laplace) best described the data and the number of times each processor (MAV/RMS) was best. Table II also tabulates "matches", i.e. records in which the best density-processor combination was either Laplace-MAV or Gauss-RMS. These matches represent records for which the experimental density is consistent with the better SNR. Table III tabulates, *by contraction level*, the density area differences (experimental vs. Gauss; experimental vs. Laplace) and the EMG amplitude estimate SNRs.

IV. Discussion

A. The Probability Density Function of Surface EMG

Overall, the results of this study indicate that the Laplace density best described these data. This result was not a function of the contraction level (see Table III), although the fit to each density improved as the contraction level increased. However, the best density clearly varied with the subject. The three "Triceps Muscle" subjects displayed densities closest to the Laplace density, while the two "Biceps Muscle" subjects displayed densities closest to the Gaussian density. Perhaps these results suggest that the density function of surface EMG differs muscle to muscle. Given the small number of subjects, however, alternative explanations (e.g. a person to person density variation) could also be hypothesized. Note that it is also possible that these results might be influenced by the electrode-amplifier configuration (particularly the inter-electrode distance). The small inter-electrode distance (15 mm) provides a very selective electrode, thereby limiting the number of motor units contributing to the observed signal. A larger number of contributing motor units (consistent with a larger inter-electrode distance), might promote an observed waveform density which is closer to Gaussian.

B. SNR Performance of RMS and MAV Processors

Both theoretic and experimental SNR results have been presented. If the data were Gaussian, then the theory predicts that RMS processor performance should exceed MAV processor performance by ≈ 6.8% (for values of *N* over the range 12-10000). Experimental results found just the opposite—MAV processing performed better, on average by 6.5%. The superiority of the MAV processor existed for all five subjects (and 73% of the individual records), although it was more pronounced in the three "Triceps Muscle" subjects.

Prior work comparing experimental SNR performance results to the Gaussian model has found that experimental SNR does increase in a square-root fashion with *N*, however

$$\text{the predicted performance level of } SNR_{MAV} = \sqrt{\left(\frac{2}{\pi-2}\right)N}$$

(for MAV processing) is never achieved. Instead, a consistently lower level of performance results. This finding is true for RMS processing as well [3]-[5]. Perhaps the

Subject	Number of Records with Density Closest in Shape to:		Number of Records in which Processor with Highest SNR is:		Number of "Matches"
	Laplace	Gauss	MAV	RMS	
AD	73	27	81	19	82 (82%)
CC	93	27	92	28	83 (69%)
DA	10	150	103	57	61 (38%)
EA	15	105	64	56	71 (59%)
FA	160	0	141	19	141 (88%)
Total	351	309	481	179	438 (66%)

Table II: Tabulated Results of Closest Density Shape and Best Processor

lower achieved performance is, in part, related to the probability density of the EMG. If the EMG density is Laplacian, then the theoretic performance (for MAV processing) is $SNR_{Laplace} = \sqrt{N}$. The Laplacian-based model performance is inferior to the Gaussian-based model performance by the fraction $\sqrt{\frac{2}{\pi-2}}$, or by approximately 32%. Thus, rather subtle changes in the probability distribution of the EMG could result in large decrements in available SNR performance.

V. Summary

The probability density function of the surface EMG was re-examined theoretically and experimentally. Theoretically, if the EMG density is Gaussian, then previous authors have shown that RMS processing is preferred. If, however, the EMG density is Laplacian, then the new model for the EMG waveform developed herein shows that MAV processing is optimal. Experimentally, it was found that the Laplacian density best fit these experimental data on average, although results varied with subject. For amplitude estimation, MAV processing was superior to RMS processing for all subjects. Combined, these results suggest that MAV processing may be preferred, both from theoretical and experimental perspectives.

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Percent MVC	Mean ± Std. Dev. Area Difference Between Experimental Density and:		Mean ± Std. Dev. SNR Using Processor:	
	Laplace	Gauss	MAV	RMS
10%	.328±.115	.376±.161	12.6±3.6	12.0±4.3
25%	.245±.049	.273±.127	11.8±2.8	11.1±3.2
50%	.198±.050	.248±.123	10.7±2.6	10.1±2.7
75%	.190±.057	.239±.116	10.4±2.5	9.7±2.4
Total	.240±.091	.284±.143	11.4±3.0	10.7±3.3

Table III: Density Area Differences and SNRs Tabulated by Contraction Level

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