Thinking About Jitter:
A Simple Method For Relating Time- and Frequency-Domain Measurements of Oscillator Stability

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Presentation Outline

Measurement Techniques ↔ Design Tools

- Applications and requirements
- Phase noise, jitter concepts
- Measurement techniques
- Noise model: IEEE-1139 standard
- Simplified noise model
- Method for relating jitter measurements
- Implications for design
- Design example: AD806 clock recovery PLL
Application and Requirements

- Serial data transmission over fiber optic link; requires:
  - Low bit error rate (BER)
  - Low cost, low power, simple interface
  - Recover bit clock from serial data
Clock Recovery with AD802 PLL

- Advantage
  - Low cost: entire system can be integrated
  - Requires integrated, low jitter VCO
Jitter Referenced to Transmit Clock

TCLK

RCLK

\[ \sigma_x \]

Data Source

Clock Recovery PLL (D.U.T)

Communications Signal Analyzer
Measurement of Jitter Referenced to TCLK
Other Applications

- Disk drive read channel
  - Same clock and data recovery task
  - Data stored on tracks without clock
  - Jitter increases read errors

- High speed digital clock distribution
  - Distribute low frequency clock
  - Use on-chip PLL to multiply to higher frequency
  - Jitter reduces timing margin
Other Applications

- Digital audio / oversampled data conversion
  - PLL used to generate multiple of fundamental sampling rate required for $\Sigma-\Delta$
  - Phase noise causes audible distortion

- Wireless communication
  - PLL used in demodulation of RF signal
  - Requires low phase noise LO
Jitter of Integrable VCOs

- LC resonant
  - Known to have best jitter performance
  - Small on-chip L restricts frequency to $\geq 10$ GHz

- Multivibrator
  - Known to have poor jitter performance in general
  - Some analysis, may be possible to improve

- Ring
  - Empirical results show good jitter performance
  - Little theoretical understanding

Need techniques for low jitter ring oscillator design
**What is Phase?**

- Phase is the argument of a trigonometric function:

\[
V(t) = V \sin \left[ \omega t + \phi \right]
\]

- Frequency is the time derivative of phase:

\[
\omega = \frac{d}{dt} \Phi(t)
\]

- Phase is the integral of frequency:

\[
\Phi(t) = \int_{\phi}^{t} \omega(t) \, dt + \phi
\]
Time Domain Phase Measurement: Jitter

- Observed Voltage
  - $V(t)$
  - $\Phi(t)$
  - Frequency $\omega(t)$

- Phase
  - $0$, $\pi$, $2\pi$, $3\pi$, $4\pi$

- Time $t$
Time Domain: Two sample standard deviation

- **Equipment**: Communications Signal Analyzer (CSA)
- **Procedure**
  - "Self referenced:" clock is both trigger and input
  - Observe distribution of delay times to threshold crossings of clock
  - Plot $\sigma$ as a function of delay $\Delta T$

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PLL VCO Time Domain Measurements

- **Open loop:**
  \[ \sigma \text{ proportional to square root of } \Delta T \]
  \[ \sigma_{\Delta T(OL)}(\Delta T) \approx K \sqrt{\Delta T} \]

- **Closed loop:**
  Action of loop limits \( \sigma \) for delays longer than loop bandwidth \( \tau \)

![Graph showing rms jitter vs. delay for open loop](image1)

- Measured jitter
- \( \kappa = 6.14E-08 \text{ } \sqrt{s} \)
- Fit to
- Predicted

![Graph showing rms jitter vs. delay for closed loop](image2)
Time Domain Measurement

- **Advantages**
  - Structure of $\sigma_{\Delta T}$ vs. $\Delta T$ plot gives information on noise process
  - Does not require access to transmit clock
  - Applicable to free running VCO (PLL open loop)

- **Disadvantages**
  - $\sigma_{\Delta T}$ fails to converge in presence of frequency drift
  - Limited by accuracy of CSA time base at long $\Delta T$
Frequency Domain Measurement: Phase Noise

IDEAL SINE WAVE      AMPLITUDE NOISE      PHASE NOISE

TIME DOMAIN

FREQUENCY DOMAIN
Frequency Domain Measurement

- Equipment: Spectrum Analyzer
- "Direct Spectrum" Procedure
  - Feed clock into RF input
  - Observe spectrum near fundamental frequency

![Diagram showing RF input and spectrum analyzer]
PLL VCO Spectrum Measurements

- Open loop:
  Integration of white noise at VCO input gives $1/f^2$ spectrum
  \[ S_{\phi_{OL}}(f) \approx \frac{N_1}{f^2} \]

- Closed loop:
  $1/f^2$ spectrum shaped by loop filter
  Spectrum rolls off for $f < f_L$
Frequency Domain Measurement

- Advantages
  - Simple and quick
  - Most work on phase noise has been done in frequency domain
  - Easy to see effect of loop filter on jitter

- Disadvantages
  - This is not how the serial data communication customer decides whether the part is working!
Relating Different Jitter Measurements

- Reason
  - Design in the domain that gives the most insight
  - Always able to relate to end user's specification

- Analysis Domains
  - Time
  - Frequency

- PLL Operating Conditions
  - Closed loop
  - Open loop (stand alone)
Measurement Technique Summary

<table>
<thead>
<tr>
<th>PLL CLOSED LOOP</th>
<th>FREQUENCY DOMAIN</th>
<th>TIME DOMAIN</th>
<th>SELF REFERENCED</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_{\Phi CL}(f)$</td>
<td>$p(t)$ with spread $\sigma_x$</td>
<td>$\sigma_{\Delta T CL}(\Delta T)$</td>
</tr>
<tr>
<td>$f_L$</td>
<td>$f$</td>
<td>$\Delta T$</td>
<td></td>
</tr>
</tbody>
</table>

| OPEN LOOP       | $S_{\Phi OL}(f)$ | $\sigma_{\Delta T OL}(\Delta T)$ |
| $N_1/f^2$       | $f$              | $\Delta T$                   |
IEEE-1139 Standard: Time Domain

- Allan variance

\[ \sigma_y^2(\tau) = \frac{1}{2} \left\langle \left( \bar{y}_{k+1} - \bar{y}_k \right)^2 \right\rangle \]

where

\[ \bar{y}_k = \frac{1}{\tau} \int_{t_k}^{t_k + \tau} y(t) \, dt = \frac{x_{k+1} - x_k}{\tau} \]
IEEE-1139 Standard: Frequency Domain

- One-sided power spectral density of phase $S_\phi(f)$
Example: Phase Noise Plot

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### IEEE-1139 Standard: Noise Process Model

#### The Functional Characteristics of the Independent Noise Processes Used in Modeling Frequency Instability of Oscillators

<table>
<thead>
<tr>
<th>Description of Noise Process</th>
<th>Frequency Domain</th>
<th>Time-Domain</th>
<th>Mod $\sigma_y(\tau)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_y(f)$ or $S_\alpha(f)$ or $S_x(f)$</td>
<td>$\alpha$</td>
<td>$\beta$</td>
<td>$\mu$</td>
</tr>
<tr>
<td>Random walk frequency modulation</td>
<td>-2</td>
<td>-4</td>
<td>1</td>
</tr>
<tr>
<td>Flicker frequency modulation</td>
<td>-1</td>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>White frequency modulation</td>
<td>0</td>
<td>-2</td>
<td>-1</td>
</tr>
<tr>
<td>Flicker phase modulation</td>
<td>1</td>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>White phase modulation</td>
<td>2</td>
<td>0</td>
<td>-2</td>
</tr>
</tbody>
</table>

\[
S_y(f) = \frac{(2\pi f)^2}{(2\pi \nu_0)^2} \quad S_\alpha(f) = h_\alpha f^\alpha
\]

\[
S_\alpha(f) = \nu_0^2 h_\alpha f^{\alpha-2} = \nu_0^2 h_\alpha f^\beta \quad (\beta \equiv \alpha - 2)
\]

\[
S_x(f) = \frac{1}{4\pi^2} h_\alpha f^{\alpha-2} = \frac{1}{4\pi^2} h_\alpha f^\beta
\]

\[
\sigma_y^2(\tau) \sim |\tau|^{\mu}
\]

\[
\sigma_y(\tau) \sim |\tau|^{\mu/2}
\]

\[
\text{Mod } \sigma_y(\tau) \sim |\tau|^{\mu'}
\]
IEEE-1139 Standard: Time/Frequency Translation

Translation of Frequency Instability Measures from Spectral Densities in Frequency Domain to Variances in Time Domain and Vice Versa (For $2\pi f_h \tau \gg 1$)

<table>
<thead>
<tr>
<th>Description of Noise Process</th>
<th>$\sigma_y^2(\tau) =$</th>
<th>$S_y(f) =$</th>
<th>$S_{\sigma}(f) =$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random walk frequency modulation</td>
<td>$A \mid f^2 S_y(f) \mid \tau^1$</td>
<td>$\frac{1}{A} [\tau^{-1} \sigma_y^2(\tau)] f^{-2}$</td>
<td>$\frac{\nu_0^2}{A} [\tau^{-1} \sigma_y^2(\tau)] f^{-4}$</td>
</tr>
<tr>
<td>Flicker frequency modulation</td>
<td>$B \mid f S_y(f) \mid \tau^0$</td>
<td>$\frac{1}{B} [\tau^{-2} \sigma_y^2(\tau)] f^{-1}$</td>
<td>$\frac{\nu_0^2}{B} [\tau^0 \sigma_y^2(\tau)] f^{-3}$</td>
</tr>
<tr>
<td>White frequency modulation</td>
<td>$C \mid f^0 S_y(f) \mid \tau^{-1}$</td>
<td>$\frac{1}{C} [\tau^1 \sigma_y^2(\tau)] f^0$</td>
<td>$\frac{\nu_0^2}{C} [\tau^1 \sigma_y^2(\tau)] f^{-2}$</td>
</tr>
<tr>
<td>Flicker phase modulation</td>
<td>$D \mid f^{-1} S_y(f) \mid \tau^{-2}$</td>
<td>$\frac{1}{D} [\tau^2 \sigma_y^2(\tau)] f^{-1}$</td>
<td>$\frac{\nu_0^2}{D} [\tau^2 \sigma_y^2(\tau)] f^{-1}$</td>
</tr>
<tr>
<td>White phase modulation</td>
<td>$E \mid f^{-2} S_y(f) \mid \tau^{-2}$</td>
<td>$\frac{1}{E} [\tau^2 \sigma_y^2(\tau)] f^2$</td>
<td>$\frac{\nu_0^2}{E} [\tau^2 \sigma_y^2(\tau)] f^0$</td>
</tr>
</tbody>
</table>

$A = \frac{4\pi^2}{6}$  
$B = 2\log_2 \frac{\epsilon}{e}$  
$C = 1/2$  
$D = \frac{1.038 + 3 \log (2\pi f_h \tau)}{4\pi^2}$  
$E = \frac{3f_h}{4\pi^2}$

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Simplified Noise Model

- Ideal VCO with white noise at input
Open Loop VCO Phase Noise

\[ \log S_{\Phi}(f) \]

\[ \log S_{V_{\text{ctl}}}(f) \]

V_{\text{ctl}} \text{ INPUT WHITE NOISE p.s.d.}

OPEN LOOP PHASE NOISE p.s.d. (INTEGRATED WHITE NOISE)

\[ \log f \]
PLL Response to Phase Noise

\[ K_d (\theta_i - \theta_o) \quad F(s) \quad \frac{K_o}{s} \quad \theta_n \rightarrow \theta_o \]

\[ \log \left| H(f) \right| \quad \log f \]

\[ \left| \frac{\theta_i}{\theta_n} \right| \quad \text{PHASE INPUT} \quad \left| \frac{\theta_o}{\theta_n} \right| \quad \text{Hn(f) PHASE NOISE TRANSFER FUNCTION} \]
Phase Noise at PLL Output

OPEN LOOP PHASE NOISE p.s.d.
(INTEGRATED WHITE NOISE)

CLOSED LOOP p.s.d.
(LOWPASS DUE TO SHAPING BY LOOP)

LOOP PHASE NOISE TRANSFER FUNCTION

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Developing Time/Frequency Translation

- Frequency domain, VCO open loop
  \[ S_{\phi OL}(f) = \frac{N_1}{f^2} \]

- Frequency domain, VCO closed loop
  \[ S_{\phi CL}(f) = \frac{N_1/f_L^2}{1 + (f/f_L)^2} \]
Developing Time/Frequency Translation

- Time domain, closed loop, transmit clock referenced

\[
\int_{-\infty}^{+\infty} S_{\phi_{CL}}(f) = \int_{-\infty}^{+\infty} \frac{N_1/f_L^2}{1 + (f/f_L)^2} = \frac{N_1 \pi}{f_L} = \sigma_x^2
\]

\[
\sigma_x = \sqrt{\frac{N_1 \pi}{f_L}} \text{ [rad rms]}
\]
Developing Time/Frequency Translation

- Time domain, closed loop, self referenced

\[ \sigma_{\Delta T(\text{CL})}^2 = 2 \left( \sigma_x^2 - R_{xx}(\Delta T) \right) \]

\[ \frac{2\tau}{1 + (2\pi ft)^2} \iff \exp(-|t|/\tau) \]

\[ R_{xx}(\Delta T) = \mathcal{F}^{-1} \left\{ \frac{N_1/f_L^2}{1 + (f/f_L)^2} \right\} = \frac{N_1 \pi}{f_L} \exp(-2\pi f_L |\Delta T|) \]

\[ \sigma_{\Delta T(\text{CL})}^2 = 2 \sigma_x^2 \left( 1 - \exp(-2\pi f_L \Delta T) \right) \]

\[ \sigma_{\Delta T(\text{CL})} = \sqrt{2} \sigma_x \sqrt{1 - \exp(-2\pi f_L \Delta T)} \]
Developing Time/Frequency Translation

- Time domain, open loop, self referenced

\[ \sigma_{\Delta T(OL)}^2 = 4 \pi \sigma_x^2 f_L \Delta T \]

\[ \sigma_{\Delta T(OL)}^2 = 4\pi^2 N_1 \Delta T \]

\[ \sigma_{\Delta T(OL)} = 2\pi \sqrt{N_1} \Delta T \text{ [rad rms]} \]

\[ \sigma_{\Delta T(OL)} = \frac{\sqrt{N_1} \Delta T}{f_0} \text{ [s rms]} \]

\[ K = \frac{\sigma_{\Delta T(OL)}}{\sqrt{\Delta T}} = \frac{\sqrt{N_1}}{f_0} \text{ [sqrt]} \]
Mathematical Relationships

**FREQUENCY DOMAIN**

<table>
<thead>
<tr>
<th>PLL CLOSED LOOP</th>
<th>OPEN LOOP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{\phi CL}(f)$</td>
<td>$S_{\phi OL}(f) = \frac{N_1}{f^2}$</td>
</tr>
<tr>
<td>$N_1/f_L^2$</td>
<td>$N_1 = K^2 f_0^2$</td>
</tr>
<tr>
<td>$1 + (f/f_L)^2$</td>
<td>$f_L$</td>
</tr>
</tbody>
</table>

**TIME DOMAIN**

<table>
<thead>
<tr>
<th>TRANSMIT CLOCK REFERENCED</th>
<th>SELF REFERENCED</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_x = \frac{1}{f_0} \sqrt{\frac{N_1}{4\pi f_L}}$</td>
<td>$\sigma_{\Delta T(OL)} = K \sqrt{\Delta T}$</td>
</tr>
<tr>
<td>$\sigma_x = K \sqrt{\frac{1}{4\pi f_L}}$</td>
<td>$\sqrt{2} \sigma_x \sqrt{1 - \exp(-2 \pi f_L \Delta T)}$</td>
</tr>
</tbody>
</table>

Mathematical relationships for PLL closed loop and open loop, including frequency domain and time domain expressions.
Benefits of Time/Frequency Relationships

- Can relate either open loop figure of merit (K or N₁) to closed loop jitter performance.
- Allows design to take place in most convenient domain
- Simplifies design and simulation: need only consider open loop VCO
- Allows stand-alone test of VCO contribution to jitter.
- Applies to any oscillator that fits 1/f^2 model

BUT...

- K and N₁ describe jitter performance of the entire oscillator: How to relate to design decisions on the level of the ring delay stage?
Gate Level Sources of Jitter

Differential pair delay stage

\[ \text{VCC} \]

\[ \text{CL1} \]
\[ \text{RL1} \]
\[ \text{Q1} \]
\[ \text{Q2} \]
\[ \text{Vout} \]
\[ \text{Vin} \]
\[ \text{VBIAS} \]
\[ \text{IEE} \]
\[ \text{VEE} \]

Delay stage with noise sources

\[ \text{VCC} \]

\[ \text{CL1} \]
\[ \text{RL1} \]
\[ \text{RL2} \]
\[ \text{en1} \]
\[ \text{en2} \]
\[ \text{Vout} \]
\[ \text{Vin} \]
\[ \text{IEE} \]
\[ \text{iEE} \]
\[ \text{VEE} \]
Collector Resistance Noise Model

\[ K_{RC} \approx (1.699) \sqrt{\frac{kT}{I_{EE}^2 R_C}} \]
### Intuitive Meaning of $\kappa$ Expressions

- **Time domain figure of merit $\kappa$:**
  - has dimensions of square root (time)
  - quantifies gate's ability to measure time accurately

- **All equations for $\kappa$ take form:**

  \[
  \sqrt{\frac{\text{UNCERTAINTY IN QUANTITY}}{\text{QUANTITY FLOW RATE}}}
  \]

- **$\kappa$ from thermal noise in collector load resistor $R_c$**

  \[
  \kappa_{R_C} \approx (1.699) \sqrt{\frac{k T}{I_{EE}^2 R_C}} \sqrt{\frac{\text{joule}}{\text{joule/sec}}}
  \]

- **$\kappa$ from shot noise in tail current $I_{EE}$**

  \[
  \kappa_{I_{EE}} \approx (0.849) \sqrt{\frac{q}{I_{EE}}} \sqrt{\frac{\text{coul}}{\text{coul/sec}}}
  \]
Schematic for Ring Experiments

- **Q1, Q2**
- **Q3, Q4**
- **R1, R2**: 500Ω
- **VCC**
- **Vin**, **Vout**
- **VCTL**
- **VBIAS**: 300 μA, 400 μA
- **VEE**: 6kΩ, 6kΩ
- **IEE**: 250Ω
Ring Experiments

- Rings of lengths 3, 4, 5, 7, and 9 stages were fabricated
- All delay stages identical

What effect does the length of the ring have on jitter?
## Measured $\kappa$ Independent of Ring Length

<table>
<thead>
<tr>
<th>RING STAGES</th>
<th>$\kappa$ [E-08/s]</th>
<th>$f_0$ [MHz]</th>
<th>$t_d$ [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4.17</td>
<td>170.1</td>
<td>980</td>
</tr>
<tr>
<td>4</td>
<td>3.56</td>
<td>164.1</td>
<td>762</td>
</tr>
<tr>
<td>5</td>
<td>3.78</td>
<td>102.7</td>
<td>974</td>
</tr>
<tr>
<td>7</td>
<td>3.77</td>
<td>71.9</td>
<td>993</td>
</tr>
<tr>
<td>9</td>
<td>3.94</td>
<td>56.8</td>
<td>978</td>
</tr>
</tbody>
</table>

- Jitter depends only on number of gates traversed, not number of oscillator periods: Length of ring is not a factor!

- Only need to consider $\kappa$ of individual gate to know jitter performance of ring
Fig. 4.20. $K$ vs. tail current
Benefits of Designing With $K$

- Can predict system level closed loop jitter as a function of circuit level parameters (resistor values, currents, etc.)
- Quick estimate of achievable jitter as a function of fundamental parameters
  - power dissipation
  - signal amplitude
- Identifies major source of jitter
- $K$ independent of:
  - number of stages in ring $N$
  - collector capacitance $C$

Allows designer freedom to manipulate $N$, $C$ to set center frequency without affecting jitter.
AD806 Measured Jitter

<table>
<thead>
<tr>
<th>Top</th>
<th>-365mV</th>
<th>Mean</th>
<th>50.09ns</th>
<th>±1σ</th>
<th>69.028%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Btm</td>
<td>-369mV</td>
<td>RMSΔ</td>
<td>43.66ps</td>
<td>±2σ</td>
<td>95.587%</td>
</tr>
<tr>
<td>Lft</td>
<td>49.85ns</td>
<td>PkPk</td>
<td>302ps</td>
<td>±3σ</td>
<td>99.69%</td>
</tr>
<tr>
<td>Stt</td>
<td>50.36ns</td>
<td>Hit #</td>
<td>2583</td>
<td>NFms</td>
<td>96</td>
</tr>
</tbody>
</table>

-365mV

Infinite Stopped

M3 Main
Summary

• Simplified noise model allows straightforward translation between time/frequency domain measurements of jitter

• Relating different jitter measurements allows design in domain that gives most insight.

• Analysis of various noise sources quantifies contributions to jitter.

• Techniques useful for many other applications requiring low jitter PLL.
Acknowledgment

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[Advanced Linear Products division]
Ring VCO Design for AD806 PLL

- Interpolating stage for voltage control of frequency
- Identical halves of ring give quadrature outputs
AD805 Ring Experiment

- Used delay stage from AD805 PLL
- Rewired stages using metal mask changes
None!

(Or at least, very little!)

Fig. 3.24. Plot of ring jitter (absolute time) for 3, 4, 5, 7, 9 stage r

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