Name ______________________________

ECE Box # ________________

Problem	Score	Points
1	_______	10
2	_______	15
3	_______	15
4	_______	25
5	_______	15
6	_______	10
7	_______	10

EE529C
Fall 2005

Noise in Analog and Mixed Signal Circuits and Systems

Exam 1

- This test is CLOSED BOOK, CLOSED NOTES. Calculator use is acceptable. A Gaussian distribution handout is appended to this exam in convenient tear-off form.
- Show **all** your work. Partial credit may be given.
- You will have three hours to complete this exam. There are 7 problems on a total of 15 pages.

1. This problem concerns the "experiment" of the Red Sox and Yankees playing a three game series. For the purposes of this problem, assume that each team is equally likely to win each game (hey, it's an exam, I can idealize the situation).

a) Show the sample space of possible outcomes for this experiment.

[2]

Events A and B are defined as follows:

A = {Red Sox win the series (win a total of 2 or 3 games) }

B = {Red Sox win the first game of the series}

b) Find the following probabilities:

[4]

The probability that the Red Sox win the series

P(A) = _____

The probability that the Red Sox win the first game

P(B) = _____

The probability that the Red Sox win the series, given that they win the first game

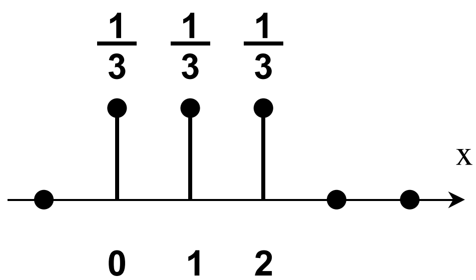
P(A|B) = _____

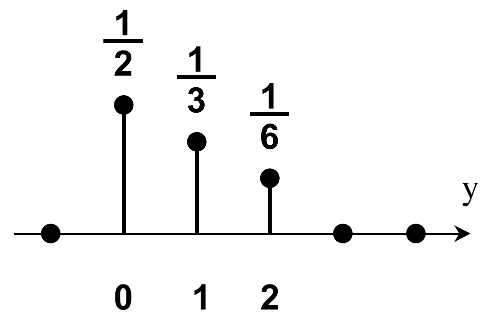
c) Are events A and B independent? Explain.

[4]

2. We are playing a betting game in which I choose (at random) one of two experiments. I then conduct a few independent trials of the experiment I chose and tell you the sequence of results. You try to guess which experiment the results came from.

The two experiments are modeled with RVs X and Y with PMFs shown below:





$$P_X(x) = \begin{cases} 1/3 & x = 0 \\ 1/3 & x = 1 \\ 1/3 & x = 2 \\ 0 & otherwise \end{cases}$$

$$P_Y(y) = \begin{cases} 1/2 & y = 0 \\ 1/3 & y = 1 \\ 1/6 & y = 2 \\ 0 & otherwise \end{cases}$$

a) Find the expected value μ and variance Var[] for each random variable:

[8]

μ_X = ________ μ_Y = ________

Var[X] = ________ Var[Y] = ________

b) After choosing an experiment at random and conducting five independent trials, my observations are

{ 0 1 0 2 0 }

[4]

What is the probability of this sequence if X was the R.V?

P = ____________

What is the probability of this sequence if Y was the R.V?

P = ____________

c) On a maximum likelihood basis, is it more probable that X or Y was the experiment chosen? Explain.

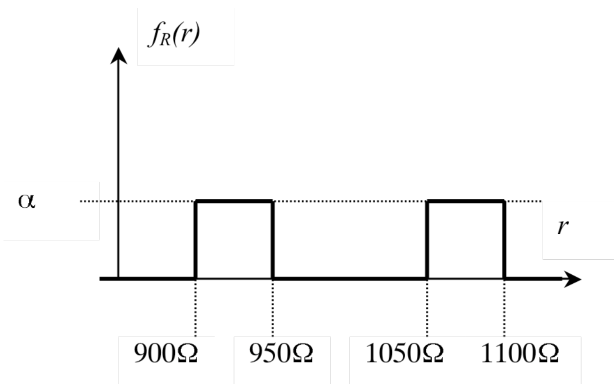
[3]

3. Did you ever wonder why nominal resistor values are in that strange sequence:

1.0, 1.2, 1.5, 1.8, 2.2, 2.7, 3.3, 3.9, 4.7, 5.6, 6.8, 8.2, 10 ?

If you look closely (and when better to take the time than on an exam) you'll see that there is approximately a 20% jump from each value to the next. That's because in the good old days, the manufacturing process for making resistors was very poorly controlled. Basically what they did was make something – anything – then measure to see what they got, and it would be within ±10% of a standard value. So in the old days, ±10% was the standard tolerance. You could get ±5% resistors, but they had to be measured and selected from the standard process flow. This had two consequences:
i) the 5% resistors were more expensive, due to the extra required measurement to identify them, and
ii) the distribution of the ±10% resistors had a big "hole" between -5% and +5% because all those resistors were selected out and sold as more expensive 5% tolerance resistors.

The result is that the pdf of (for example) 1kΩ, 10% resistors looked like Figure 2a below:



$$f_R(r) = \begin{cases} 0 & r < 900 \\ \alpha & 900 < r < 950 \\ 0 & 950 < r < 1050 \\ \alpha & 1050 < r < 1100 \\ 0 & 1100 < r \end{cases}$$

Figure 2a.

$F_R(r) =$

Figure 2b.

a) Find the value of the parameter α such that $f_R(r)$ is a valid density function.

α = __________ **[3]**

b) Find the mean (expected value) E[**R**]

E[**R**] = __________ **[3]**

c) Find the variance Var[**R**] and standard deviation σ_R (in the correct units!)

Var[**R**] = __________ σ_R = __________ **[3]**

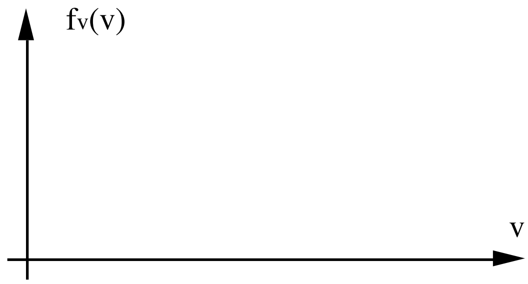
d) Find the distribution function $F_R(r)$. Plot it on the graph in Figure 2b (be sure to indicate numerical values of important points!), and write the function description in the space indicated.

[3]

e) Find the probability that the resistor value exceeds 930Ω

P[**R** > 930Ω] = __________ **[3]**

4. Battery cells with a nominal voltage of 1.5V are produced in a manufacturing process. The actual voltage is a random variable **V** which is uniformly distributed between 1.4V and 1.6V.

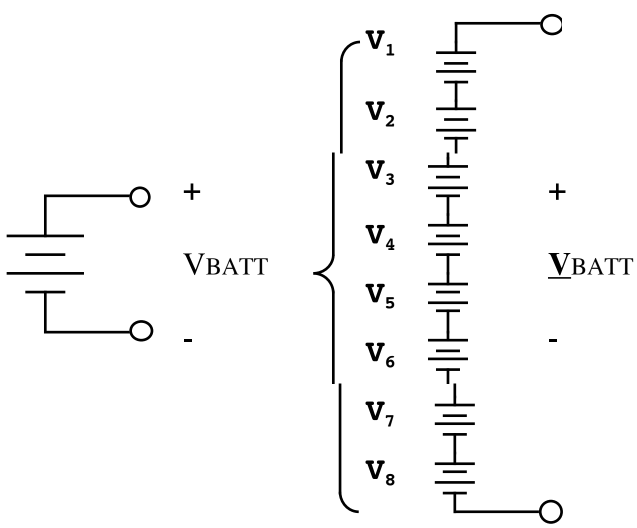


a) On the graph above, show the probability density function $f_v(v)$. Be sure to label all significant values on your plot! **[3]**

b) Determine the probability that the cell voltage will be within 2% of its nominal value: **[3]**

$P\{1.47V < \mathbf{V} < 1.53V\}$ = ________

To make a 12V (nominal) battery, eight cells are stacked in series as shown below. You may assume the cell voltages $\mathbf{V}_1, \ldots, \mathbf{V}_8$ to be independent identically distributed random variables.



c) Find the mean and standard deviation of $\mathbf{V}_{BATT}$: **[8]**

μ_{VBATT} = ________ σ_{VBATT} = ________

d) In the space below, sketch the approximate pdf of V_{BATT}:

[3]

e) Estimate the probability that the voltage $\mathbf{V_{BATT}}$ will be within 2% of its nominal value:

[5]

$P\{11.76V < \mathbf{V_{BATT}} < 12.24V\}$ = ________

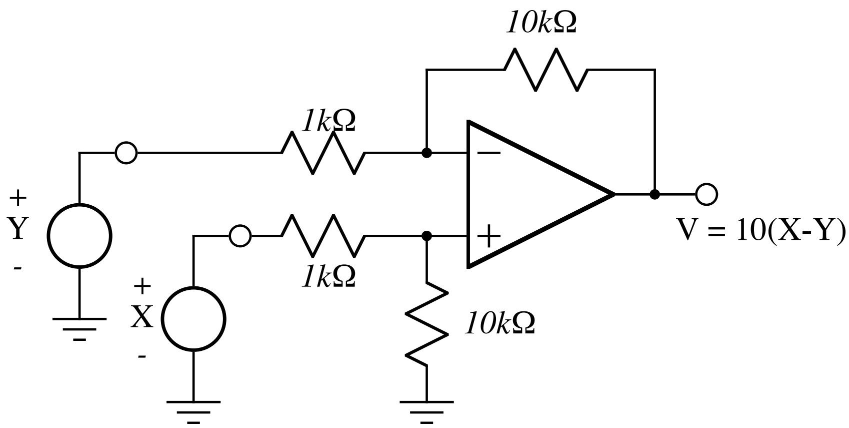
f) Due to problems with a supplier, a new batch of 1.5V cells comes in which are uniformly distributed from 1.35V to 1.55V.

With this new batch, estimate the probability that the voltage $\mathbf{V_{BATT}}$ will be within 2% of its nominal value:

[3]

$P\{11.76V < \mathbf{V_{BATT}} < 12.24V\}$ = ________

5. The circuit below is used in a biomedical application.



X and Y are independent Gaussian random variables with

X : μ_X = 5.0 mV σ_X = 2.0mV

Y : μ_Y = 4.9 mV σ_Y = 0.5mV

The circuit forms an output random variable **Z** = 10(**X-Y**) .

[12]

a) Find the mean and standard deviation of **Z**

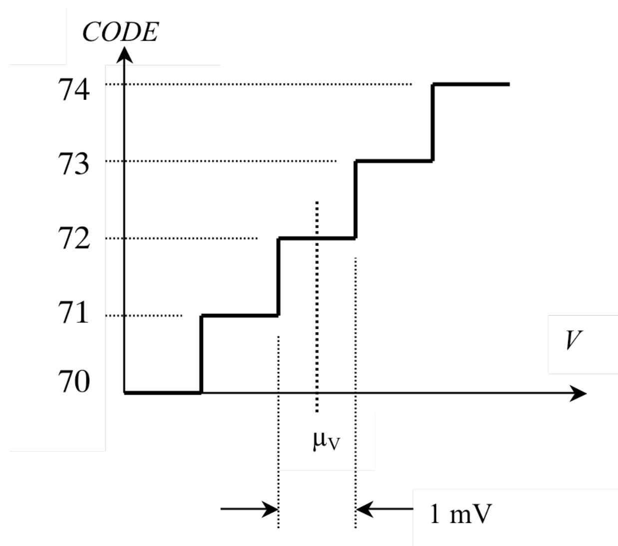
μ_Z = _________ σ_Z = _________

b) Find the probability that the output **Z** is in the range 0 to 10mV

[3]

P[0 < **Z** < 10mV] = ___________

6. An ADC has an LSB step size of 1mV. The input voltage to the ADC is a Gaussian random variable **V** with a mean at the center of code 72.



You compile statistics on 1000 independent samples at the ADC output and find the following distribution of "code hits" (samples):

CODE	# of hits
70	0
71	7
72	986
73	7
74	0

Estimate the standard deviation of V

[10]

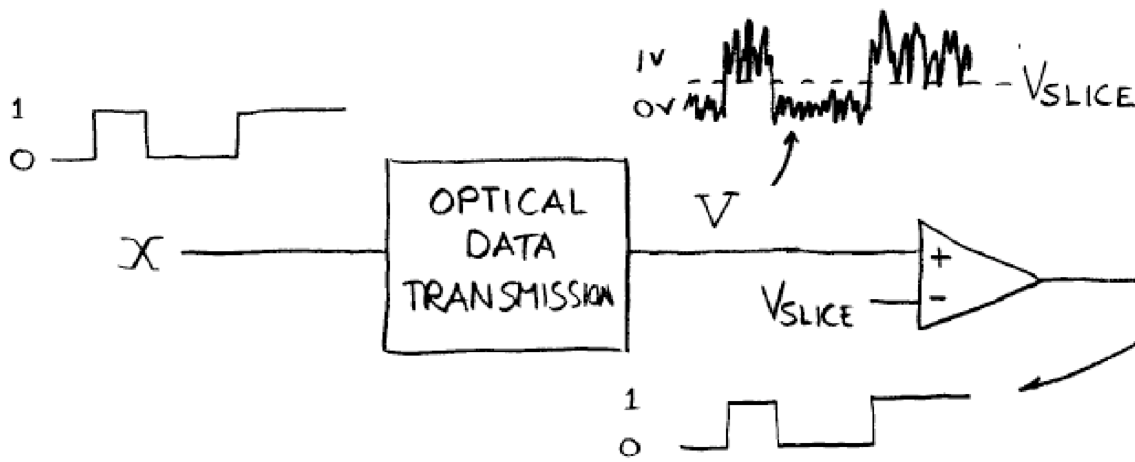
σ_V = _________

7. In an optical data transmission system, the input is a data source represented by a discrete random variable **X** which is equally likely to be a zero or one. The output of the data tranmission system is a voltage, represented as continuous random variable **V**.

The system encoding is such that, in the absence of noise:
an input of X=0 corresponds to an output voltage of V=0V, and
an input of X=1 corresponds to an output voltage of V=+1V.

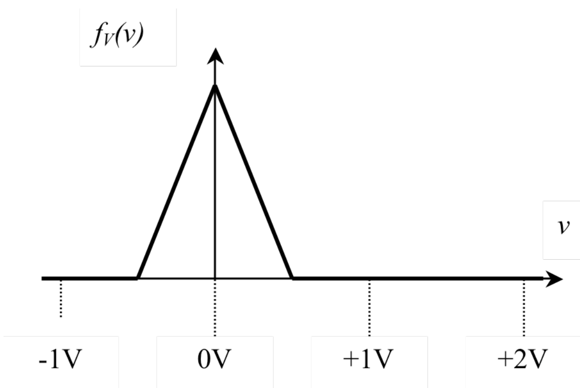
At the receiving end, a "data slicer" (comparator) compares the value of V to a slicing threshold voltage V_{SLICE}:
If $V > V_{SLICE}$, the system output Y is equal to 1.
If $V \le V_{SLICE}$, the system output Y is equal to 0.

Originally the slicing level VSLICE is set at 0.5V, midway between the two ideal output voltages.



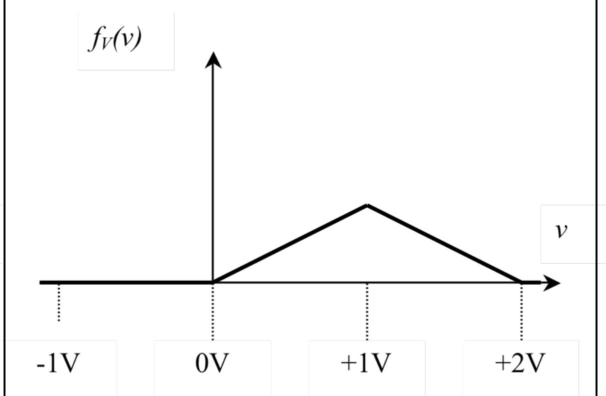
As it turns out, the randomness (noise) is dependent on the signal level.

pdf of V
when X =0



$$f_V(v) = \begin{cases} 0 & v < -0.5 \\ 2+4v & -0.5 < v < 0 \\ 2-4v & 0 < v < +0.5 \\ 0 & +0.5 < v \end{cases}$$

pdf of V
when X=1



$$f_V(v) = \begin{cases} 0 & v < 0 \\ v & 0 < v < 1 \\ 2-v & 1 < v < 2 \\ 0 & 2 < v \end{cases}$$

a) Determine the probability of an error when X=0

P[Y=1 | X=0] = ______________

b) Determine the probability of an error when X=1

P[Y=0 | X=1] = ______________

b) Given that X is equally likely to be 0 or 1, determine the probability of an error

P[error] = ______________

c) Determine the optimal value of $V_{SLICE(opt)}$ that minimizes P[error]

$V_{SLICE(opt)}$ = ______________