Studio 8 & 9 Review

- Operational Amplifier
  - Stability
  - Compensation
  - Miller Effect
  - Phase Margin
  - Unity Gain Frequency
  - Slew Rate Limiting

- Reading: Razavi ch. 9, 10
  - Lab 8, 9 op-amp is Fig. 10.34 in sec. 10.5.1
  - (see also Johns & Martin sec 5.2 pp. 232-242)
Two-stage op-amp

\[ V_{DD} = +5V \]

All P: \( \frac{900}{10} \)

All N: \( \frac{350}{10} \)

\[ V_{SS} = -5V \]

\[ 50\mu A \]

\[ V_{G6} \]

\[ V_{S1} \]
Analysis Strategy

• Recognize sub-blocks
• Represent as cascade of simple stages

V_{DD} = +5V

V_{SS} = -5V

M1, M2, M3, M4, M5, M6, M7, M8

V_{G3}, V_{G4}, V_{G5}, V_{G6}

V_{1}, V_{i2}, V_{S1}

All P: \frac{900}{10}
All N: \frac{350}{10}

50\mu A
Total op-amp model

Input differential pair    Common source stage
## DC operating point

<table>
<thead>
<tr>
<th></th>
<th>$I_D[\mu A]$</th>
<th>$V_{GS-TH}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>25</td>
<td>0.235</td>
</tr>
<tr>
<td>M2</td>
<td>25</td>
<td>0.235</td>
</tr>
<tr>
<td>M3</td>
<td>25</td>
<td>0.247</td>
</tr>
<tr>
<td>M4</td>
<td>25</td>
<td>0.247</td>
</tr>
<tr>
<td>M5</td>
<td>50</td>
<td>0.350</td>
</tr>
<tr>
<td>M6</td>
<td>50</td>
<td>0.332</td>
</tr>
<tr>
<td>M7</td>
<td>50</td>
<td>0.332</td>
</tr>
<tr>
<td>M8</td>
<td>50</td>
<td>0.332</td>
</tr>
</tbody>
</table>
# Small signal parameters

<table>
<thead>
<tr>
<th></th>
<th>$I_D$ [$\mu$A]</th>
<th>$V_{GS-V_{TH}}$</th>
<th>$g_m$ [$\mu$A/V]</th>
<th>$r_O$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>25</td>
<td>0.235</td>
<td>208</td>
<td></td>
</tr>
<tr>
<td>M2</td>
<td>25</td>
<td>0.235</td>
<td></td>
<td>800kΩ</td>
</tr>
<tr>
<td>M3</td>
<td>25</td>
<td>0.247</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M4</td>
<td>25</td>
<td>0.247</td>
<td></td>
<td>1.43MΩ</td>
</tr>
<tr>
<td>M5</td>
<td>50</td>
<td>0.350</td>
<td>285</td>
<td>715kΩ</td>
</tr>
<tr>
<td>M6</td>
<td>50</td>
<td>0.332</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M7</td>
<td>50</td>
<td>0.332</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M8</td>
<td>50</td>
<td>0.332</td>
<td></td>
<td>400kΩ</td>
</tr>
</tbody>
</table>

Note: $\lambda_n = 0.050$ V$^{-1}$; $\lambda_p = 0.028$ V$^{-1}$
Total op-amp model: Low frequency gain

Input differential pair
\[ a_{v1} = g_m (r_{o2} \| r_{o4}) \]
\[ a_{v1} = (208 \mu A/V)(800 k\Omega \| 1.43 M\Omega) \]
\[ a_{v1} = 106 \]

Common source stage
\[ a_{v2} = g_m (r_{o5} \| r_{o8}) \]
\[ a_{v2} = (285 \mu A/V)(400 k\Omega \| 715 k\Omega) \]
\[ a_{v2} = 73 \]
Total op-amp model with capacitances

Gate of M5

\[ C_g = (900\, \mu m)(10\, \mu m) \left(4.17E - 4\frac{F}{m^2}\right) \]

\[ C_g = 3.74\, pF \]

Load: scope probe \( \approx 10\, pF \)
Total op-amp model with capacitances

First stage pole

\[ f_{p1} = \frac{1}{2\pi (r_{O2}\parallel r_{O4})C_{g5}} \]

\[ f_{p1} = \frac{1}{2\pi (800k\Omega\parallel 1.43M\Omega)(3.74\ pF)} \]

\[ f_{p1} = 82kHz \]

Second stage pole

\[ f_{p1} = \frac{1}{2\pi (r_{O5}\parallel r_{O8})C_L} \]

\[ f_{p1} = \frac{1}{2\pi (400k\Omega\parallel 715k\Omega)(10\ pF)} \]

\[ f_{p1} = 61kHz \]
Open loop transfer function

• Product of individual stage transfer functions

\[
A(j\omega) = \frac{g_{m1}(r_{o2}|r_{o4})g_{m5}(r_{o5}|r_{o8})}{1 + j\omega(r_{o2}|r_{o4})C_{g5}} \left[ 1 + j\omega(r_{o5}|r_{o8})C_L \right]
\]

• Numerically (using \( \omega = 2\pi f \))

\[
A(j\omega) = \frac{7738}{1 + j\left(\frac{f}{82kHz}\right)} \left[ 1 + j\left(\frac{f}{61kHz}\right) \right]
\]

• Check Bode plot simulation; predicts:
  – DC gain = 20\log(7738) = +78dB
  – Unity gain frequency ~ 6.2 MHz
Two-stage op-amp: Simulation Schematic
DC Operating Point Simulation

DC Response

OP POINT 2.885 mV

Systematic Offset!

A: (2.11783m -3.81929)  deltA: (1.39772m 7.02858)
B: (3.51555m 3.20929)  slope: 5.02881K
Bode plot

- Magnitude, phase on log scales
- Pole: Root of denominator polynomial

SLOPE:
-20dB/dec
Open loop Bode plot

- **Product of terms**: Sum on log-log plot

![Open loop Bode plot diagram](image)
Open Loop Bode Plot Simulation

Note: AC source at input also needs DC component to account for systematic offset!
Check Open Loop Bode Plot Simulation

√DC gain ~ +78dB

Unity gain ~ 16MHz

A: (15.592M, 36.0451m)  delta: (255.918k, -195.979)
B: (15.8489M, -198.942)  slope: -766.698u
Stability example: Closed loop follower

• Negative feedback: Output connected to inverting input
• Gain should be \( \sim 1 \)

\[
\begin{align*}
\nu_{\text{out}} &= A(\nu_{\text{in}} - \nu_{\text{out}}) \\
\nu_{\text{out}}(A + 1) &= A\nu_{\text{in}} \\
\nu_{\text{out}} &= \left( \frac{A}{A + 1} \right) \nu_{\text{in}} \\
&\approx 1 \text{ as } A \gg 1
\end{align*}
\]
Unity gain: Why bother?

- No buffer:
  Voltage divider
- Signal reduced due to voltage drop across $R_S$

- With buffer:
  No current required from source

$v_{out} = \left( \frac{R_L}{R_L + R_S} \right) v_{in}$

$v_{out} = v_{in}$
Lab 9 Problem: Instability

- Oscillation superimposed on desired output!?!
Lab 9 Problem: Instability

- Ground \( v_{in} \): Output for zero input?!?
- Why? Need...
Controls: ES3011 in 20 minutes

• General framework
  A: Forward Gain
  $\beta$: Feedback Factor
  fraction of output fed back to input

![Diagram of feedback control system]
Example: Op-amp, Noninverting Gain

A: Forward Gain
Op-amp open loop gain
\[ V_{out} = A(V_+ - V_-) \]
Transfer function \( A(j\omega) \)

\[ \beta: \text{Feedback Factor} \]
\[ \beta = \frac{R_1}{R_1 + R_2} \]
Closed Loop Gain

- **Output**
  \[ v_{out} = A\left(v_{in} - \beta v_{out}\right) \]
  \[ v_{+} - v_{-} \]

- **Solve for** \( \frac{v_{out}}{v_{in}} \)
  \[ v_{out} = Av_{in} - A\beta v_{out} \]
  \[ (1 + A\beta)v_{out} = Av_{in} \]
  \[ \frac{v_{out}}{v_{in}} = \frac{A}{1 + A\beta} \]
Op-amp with negative feedback

• If $A\beta >> 1$

$$\frac{v_{out}}{v_{in}} = \frac{A}{1 + A\beta} \approx \frac{A}{A\beta} \Rightarrow \frac{v_{out}}{v_{in}} \approx \frac{1}{\beta}$$

• Closed loop gain determined only by $\beta$

• Advantage of negative feedback:
  Open loop gain $A$ can be ugly (nonlinear, poorly controlled) as long as it's large!
Example: Op-amp, Noninverting Gain

\( \beta: \text{Feedback Factor} \)

\[ \beta = \frac{R_1}{R_1 + R_2} \]

Closed loop gain

\[ \frac{v_{\text{out}}}{v_{\text{in}}} = \frac{R_1 + R_2}{R_1} = \frac{1}{\beta} \]
Reexamine closed loop transfer function

- Output with no input: infinite gain
- Infinite when $1 + A\beta = 0$
- Condition for oscillation:
  \[
  1 + A\beta = 0
  \]
- In general $A, \beta$ functions of $\omega$
- If there's a frequency $\omega$ at which $1 + A\beta = 0$: Oscillation at that frequency!
Example: follower

\[ \beta = 1 \quad \rightarrow \quad \frac{v_{out}}{v_{in}} = \frac{A}{1 + A} \]

- Use \( A(j\omega) \),
  solve for \( 1+A = 0 \)
- No thanks!

\[ A(j\omega) = \frac{g_{m1}(r_{o2}\parallel r_{o4})g_{m5}(r_{o5}\parallel r_{o8})}{\left[1 + j\omega(r_{o2}\parallel r_{o4})C_{g5}\right]\left[1 + j\omega(r_{o5}\parallel r_{o8})C_L\right]} \]
Reexamine condition for oscillation

\[ 1 + A\beta = 0 \rightarrow A\beta = -1 \]

Magnitude and phase condition:

\[ |A\beta| = 1 \text{ AND } \angle A\beta = -180^\circ \]

- Easier to get from Bode plot
Look at original $A\beta$ for 2 stage op-amp

- Find $\omega$ at which $|A\beta| = 1$; Check $\angle A\beta \ -180^\circ$?

Trouble!
Simulation Aβ for 2 stage op-amp

AC Response

Unity loop gain at ~ 16MHz

> 180° phase lag at unity loop gain!

- Causes closed-loop instability
Compensation: “Dominant Pole”

- Move one pole to lower frequency
- How?

Move unity loop gain frequency $f_T$ to lower value
So accumulated phase lag at $f_T$ hasn’t reached -180°
Compensation: “Dominant Pole”

- Need to increase capacitance by $\approx 1000X$:
  BAD! Die area cost
Miller Effect

- Impedance across inverting gain stage $G$
- Reduced by factor equal to $(1+G)$
Math for Miller effect

\[ i_x = \frac{v_x - (-Gv_x)}{Z} \]

\[ i_x = \frac{v_x(1 + G)}{Z} \]

\[ \frac{v_x}{i_x} = Z_{in} = \frac{Z}{(1 + G)} \]

- Impedance across inverting gain stage \( G \)
- Reduced by factor equal to \((1+G)\)
Example: Impedance is capacitive

- Capacitance multiplied by (1+G)
  \[ Z_{in} = \frac{Z}{(1 + G)} \]
  \[ Z = \frac{1}{sC} \quad \Rightarrow \quad Z_{in} = \frac{1}{s(1 + G)C} \]
  
  - Equivalent capacitance higher by factor 1+G
  - Problem for high bandwidth amplifiers
  - Opportunity for compensation ...
Miller Compensation

• Need effect of large capacitance
• Use Miller effect to multiply small on-chip capacitance to higher effective value
• Effect of large capacitance without die area cost of large capacitance
New schematic

- Add $C_C$ across 2nd stage
New loop gain transfer function

AC Response

Unity loop gain at ~65kHz

125° phase lag at unity loop gain
New step response

• No oscillation!
New step response with $C_C$

- Zoom in on small-signal step response:
  Some overshoot and ringing
Reason: RHP zero in complete transfer function

Complete transfer function looks like:

\[
A(j\omega) = \frac{A_0 \left[1 - j(\omega/\omega_Z)\right]}{1 + j(\omega/\omega_{p1})\left[1 + j(\omega/\omega_{p2})\right]}
\]

See Razavi 10.5, Johns & Martin 5.2
"Phase margin"

- How stable is new transfer function?
- Phase margin = Phase lag at $|A\beta| = 1$ minus (-180°)
- Usually want at least 60° for stable step response
Phase margin of op-amp with $C_C$

Unity loop gain at ~65kHz

125° phase lag at unity loop gain

Phase margin = 55°
Solution to RHP zero problem

- Add $R_Z$ in series with $C_C$

Moves RHP zero to much higher frequency
New step response with $R_Z$, $C_C$

- Zoom in on small-signal step response:
  No overshoot, ringing: phase margin improved
Large signal step response

- Slew Rate Limiting!?!?

See Solomon op-amp paper for model; rising/falling asymmetry
Dominant pole op-amp model

- Simpler model with dominant pole from $C_C$
Approximate dominant pole transfer function

\[ |A(j\omega)| \approx \frac{g_{m1}(r_{o2}\|r_{o4})A_2}{1 + j\omega(r_{o2}\|r_{o4})A_2C_C} \]

\[ A_2 = g_{m5}(r_{o5}\|r_{o8}) \]

Miller multiplied \( C_C \)

2\textsuperscript{nd} stage gain
Unity gain frequency

- Depends only on
  - Input stage transconductance \( g_{m1} \)
  - Compensation capacitor \( C_C \)

\[
|A(j\omega)| \approx \frac{g_{m1}(r_{o2}||r_{o4})A_2}{\omega(r_{o2}||r_{o4})A_2C_C}
\]

\[
|A(j\omega)| = 1 \quad \text{at} \quad \omega_T
\]

\[
\omega_T \approx \frac{g_{m1}}{C_C}
\]
Slew rate

- \( I = C \frac{dV}{dt} \)
- Only limited current \( I_{\text{BIAS}} \) available to charge, discharge \( C_C \)
Slew rate

\[ I = C \frac{dV}{dt} \implies \frac{dV}{dt} = \frac{I_{BIAS}}{C_C} \]
Summary Op-amp:

- Stability
- Compensation
- Miller effect
- Phase Margin
- Unity gain frequency
- Slew Rate Limiting