Radar Systems Engineering
Lecture 7 – Part 1
Radar Cross Section

Dr. Robert M. O'Donnell
IEEE New Hampshire Section
Guest Lecturer

Block Diagram of Radar System

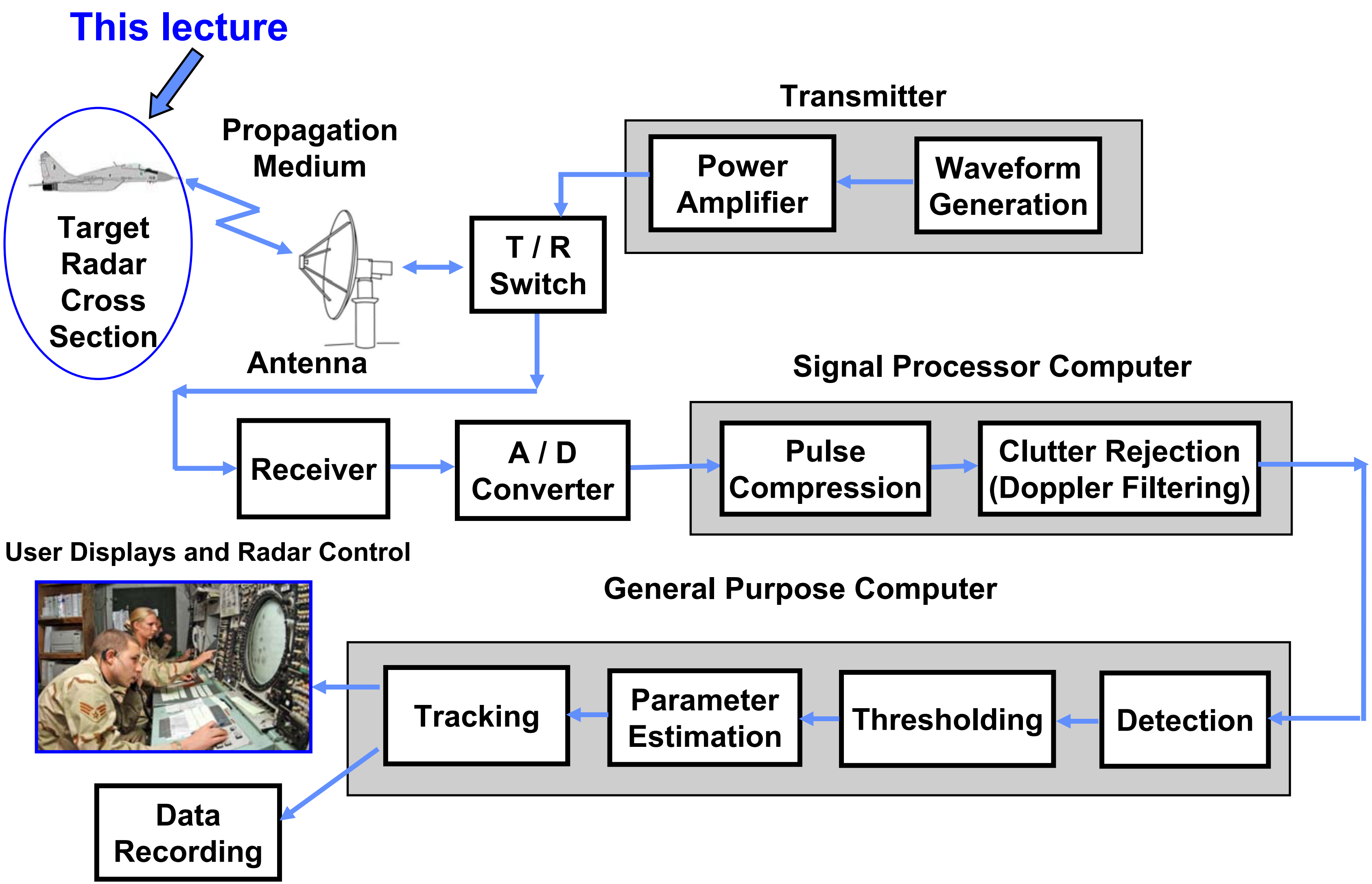

Photo Image
Courtesy of US Air Force
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Definition - Radar Cross Section (RCS or σ)

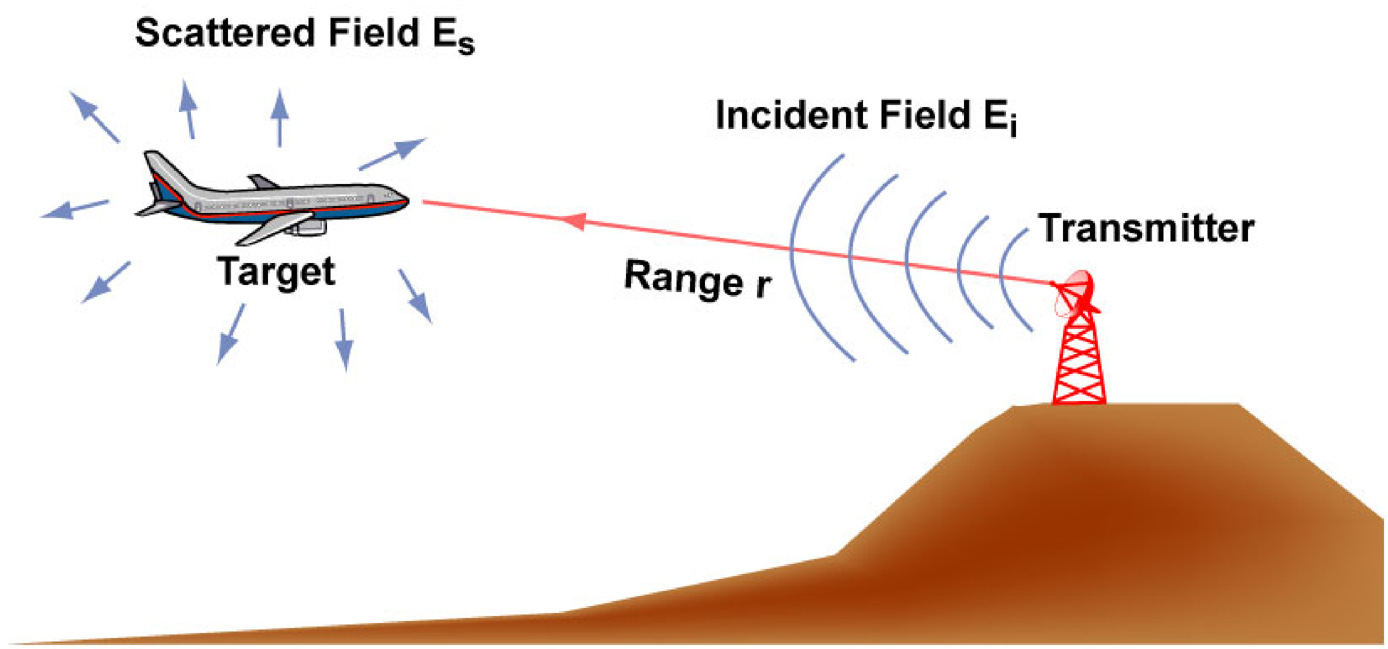


Figure by MIT OCW.

$$RCS = \lim_{r \to \infty} 4 \pi r^2 \frac{|E_s|^2}{|E_i|^2} \quad \text{(Unit: Area)}$$

Radar Cross Section (RCS) is the hypothetical area, that would intercept the incident power at the target, which if scattered isotropically, would produce the same echo power at the radar, as the actual target.

Factors Determining RCS

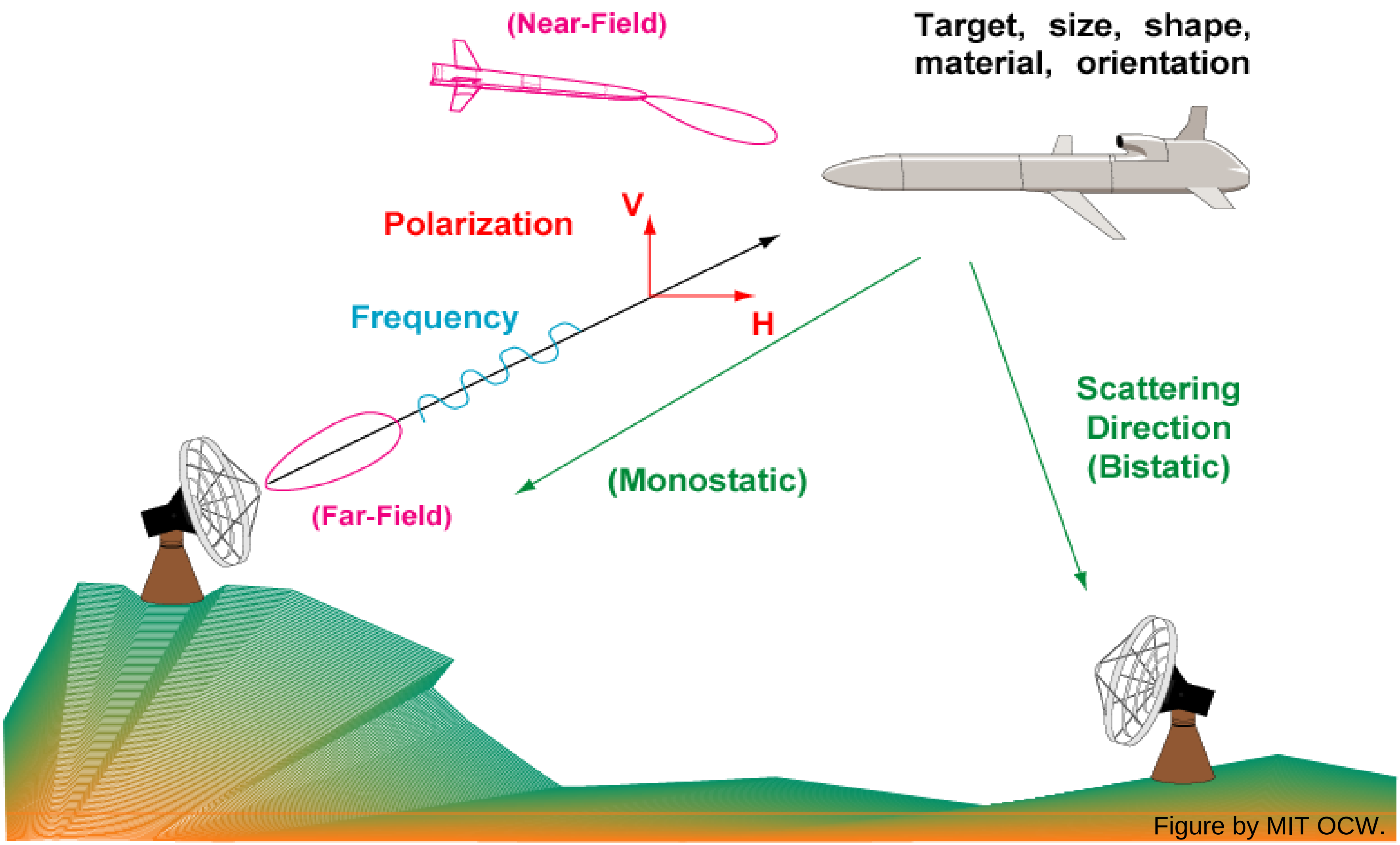


Figure by MIT OCW.

Threat's View of the Radar Range Equation

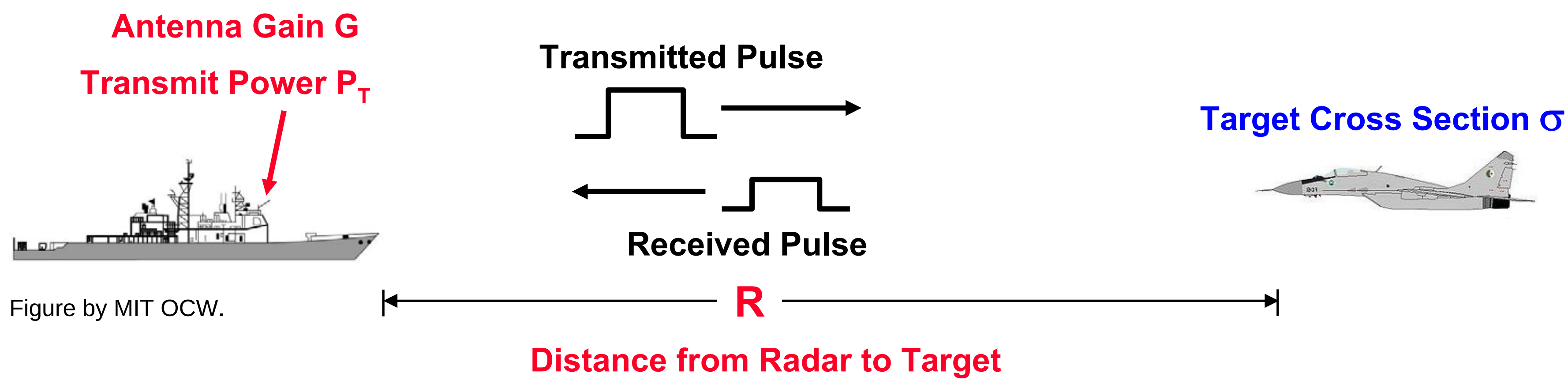


Figure by MIT OCW.

Radar Range Equation

Cannot Control → $P_t G^2 \lambda^2$

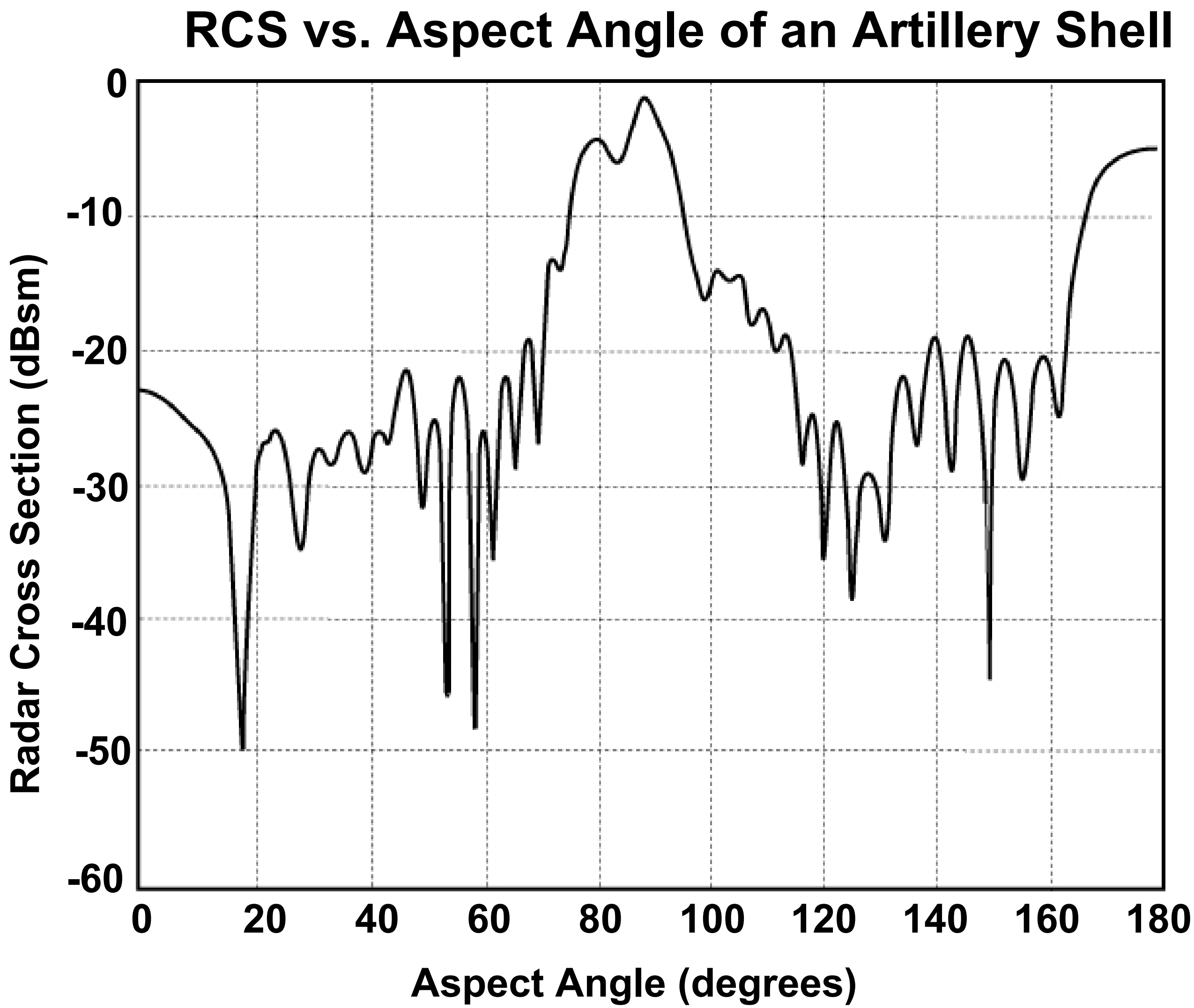
Can Control → σ

$$\frac{S}{N} = \frac{P_t \, G^2 \, \lambda^2 \, \sigma}{(4\pi)^3 \, R^4 \, k \, T_S \, B_n \, L}$$

Cannot Control → $(4\pi)^3 R^4 k T_S B_n L$

Outline

- Radar cross section (RCS) of typical targets
  - Variation with frequency, type of target, etc.
- Physical scattering mechanisms and contributors to the RCS of a target
- Prediction of a target's radar cross section
  - Measurement
  - Theoretical Calculation



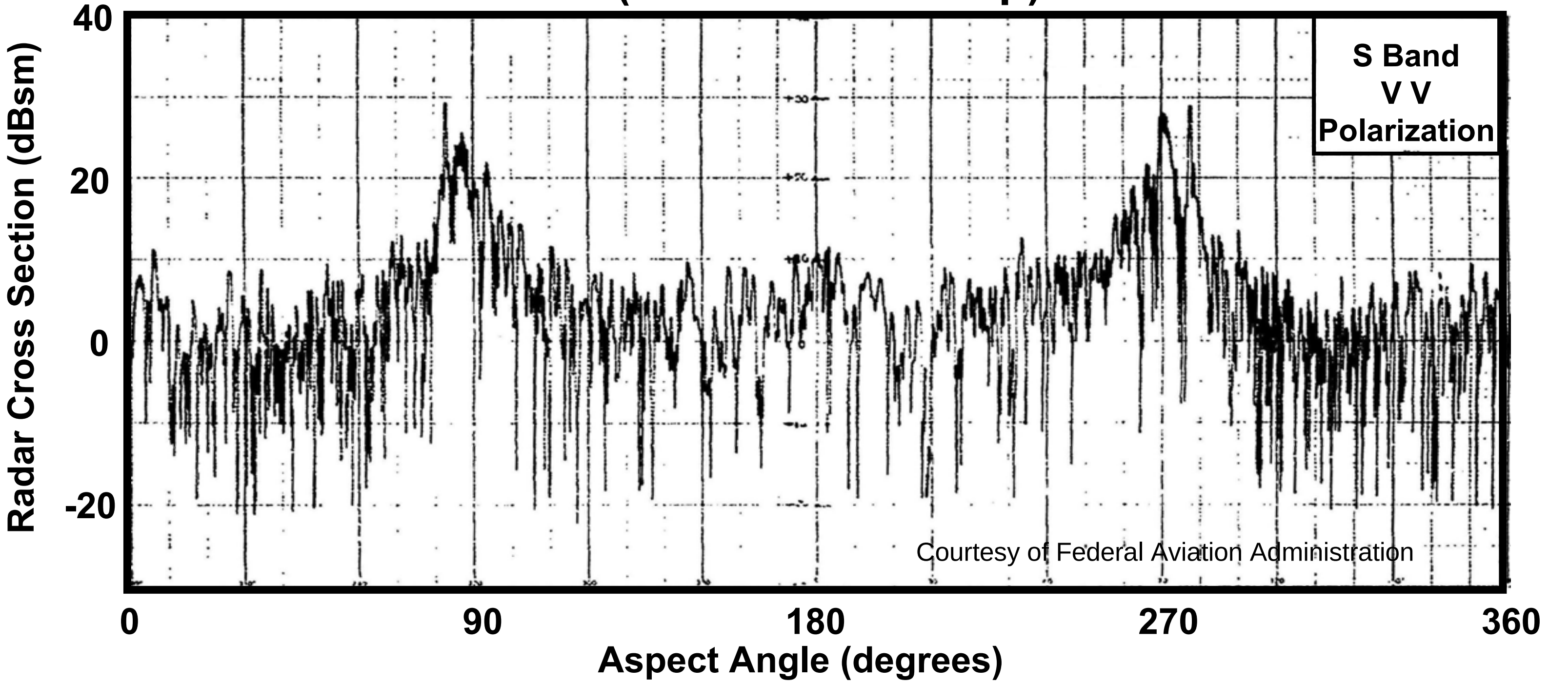
Typical Artillery Shell

Courtesy US Marine Corps

M107 Shell for 155mm Howitzer

Radar Cross Section of Cessna 150L

Measured at RATSCAT (6585th Test Group) Holloman AFB for FAA



Cessna 150L (in takeoff)

Cessna 150L (in flight)

Aspect Angle Dependence of RCS

Cone Sphere Re-entry Vehicle (RV) Example

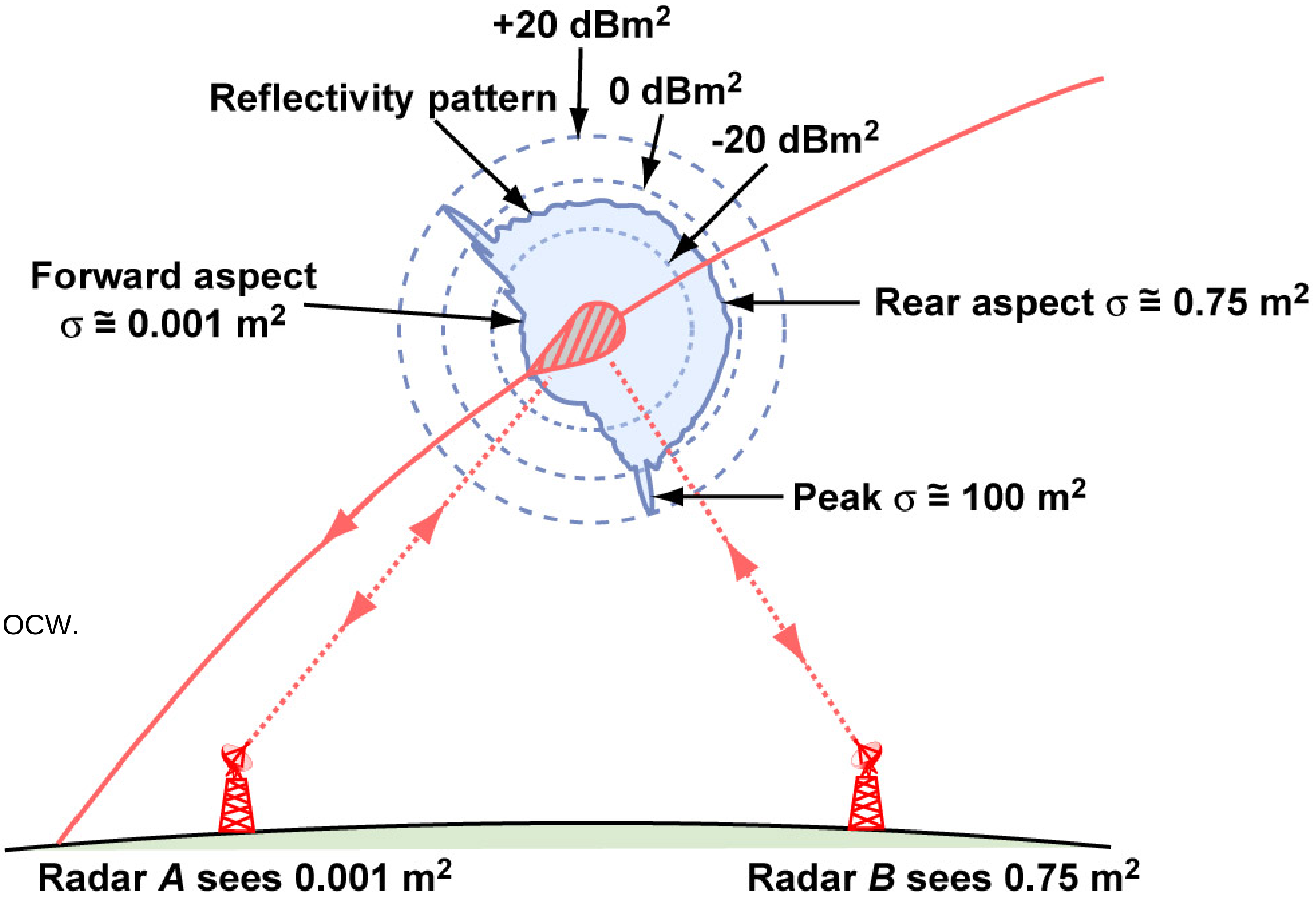


Figure by MIT OCW.

Examples of Radar Cross Sections

	Square meters
Conventional winged missile	0.1
Small, single engine aircraft, or jet fighter	1
Four passenger jet	2
Large fighter	6
Medium jet airliner	40
Jumbo jet	100
Helicopter	3
Small open boat	0.02
Small pleasure boat (20-30 ft)	2
Cabin cruiser (40-50 ft)	10
Ship (5,000 tons displacement, L Band)	10,000
Automobile / Small truck	100 - 200
Bicycle	2
Man	1
Birds (large -> medium)	10^{-2} - 10^{-3}
Insects (locust -> fly)	10^{-4} - 10^{-5}

Adapted from Skolnik, Reference 2

Radar Cross Sections of Targets Span at least 50 dB

Outline

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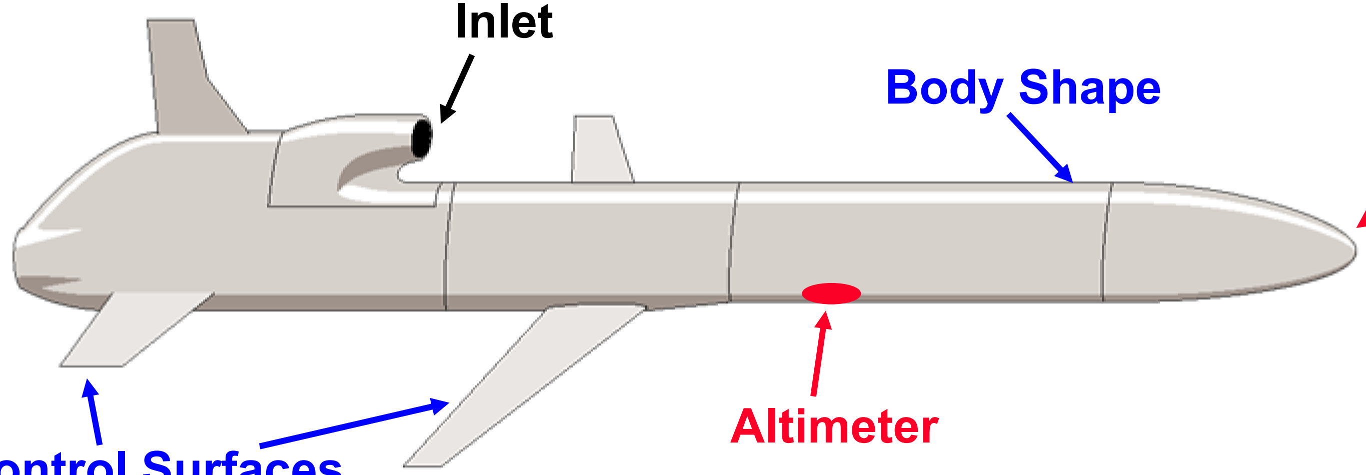
RCS Target Contributors

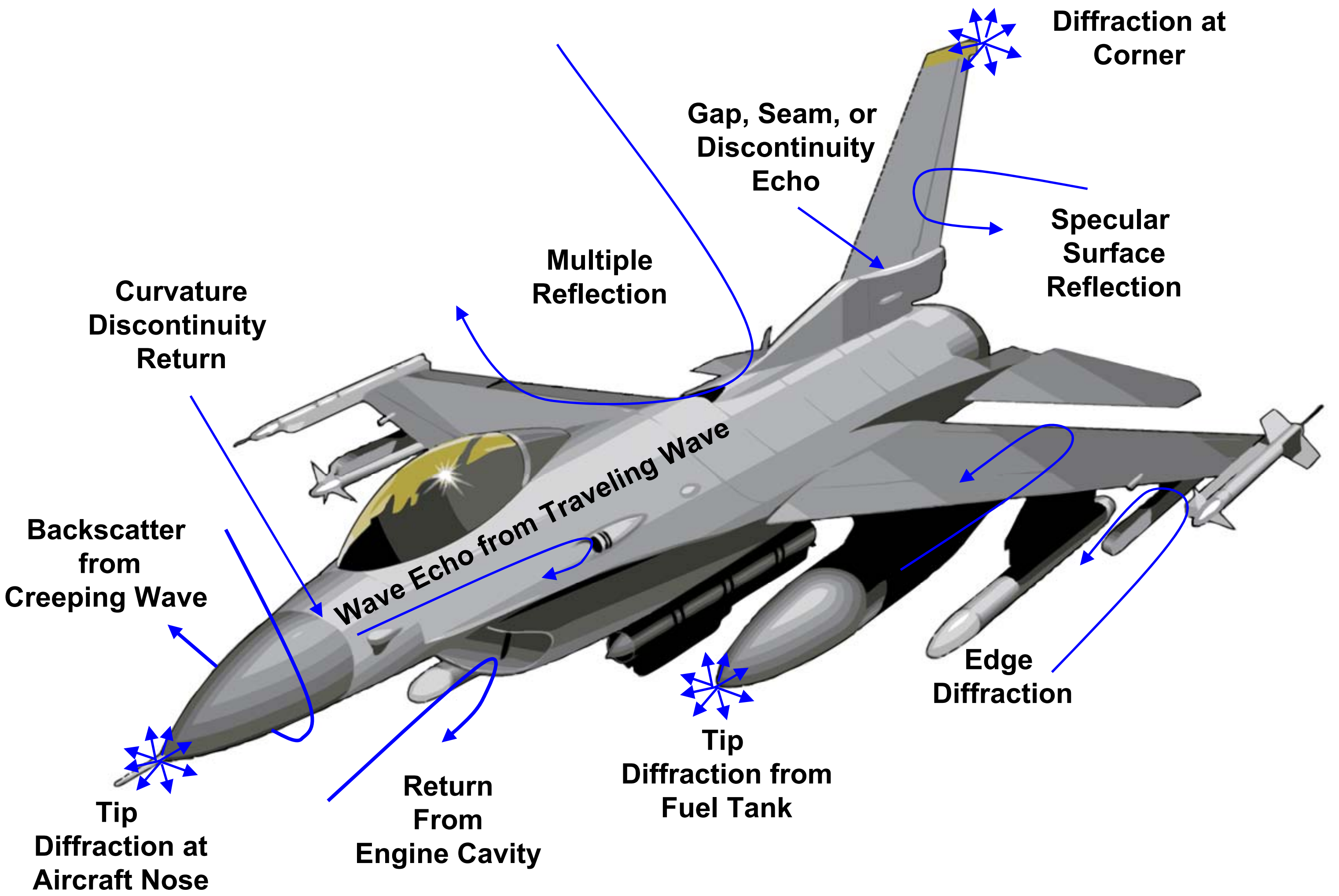


- Types of RCS Contributors
  - Structural (Body shape, Control surfaces, etc.)
  - Avionics (Altimeter, Seeker, GPS, etc.)
  - Propulsion (Engine inlets and exhausts, etc.)

Single and Multiple Frequency RCS Calculations with the FD-FD Technique

- **RCS Calculations for a Single Frequency**
  - Illuminate target with incident sinusoidal wave
  - Sequentially in time, update the electric and magnetic fields, until steady state conditions are met
  - The scattered wave's amplitude and phase can the be calculated
- **RCS Calculations for a Multiple Frequencies**
  - Illuminate target with incident Gaussian pulse
  - Calculate the transient response
  - Calculate to Fourier transforms of both:
    - Incident Gaussian pulse, and
    - Transient response
  - RCS at multiple frequencies is calculated from the ratios of these two quantities

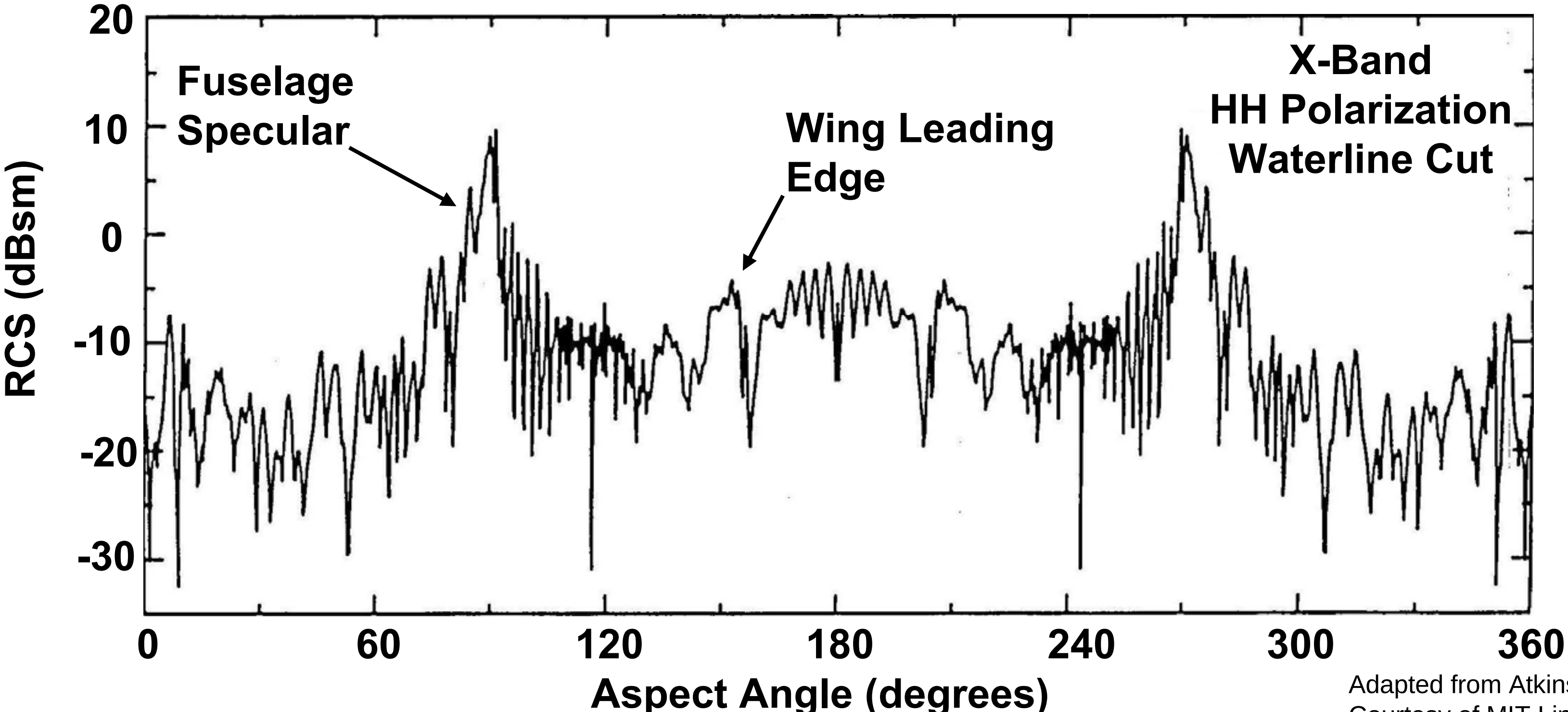
Scattering Mechanisms for an Arbitrary Target



Measured RCS of C-29 Aircraft Model

1/12 Scale Model Measurement

Full Scale C-29
BAE Hawker 125-800



Adapted from Atkins, Reference 5
Courtesy of MIT Lincoln Laboratory

Outline

- Radar cross section (RCS) of typical targets
  – Variation with frequency, type of target, etc.

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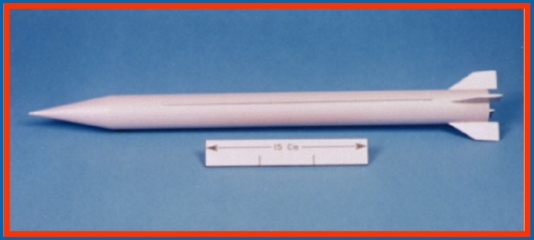
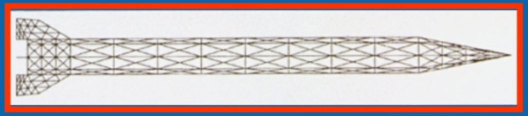
  – Measurement
  – Theoretical Calculation

Techniques for RCS Analysis

Full Scale Measurements





Theoretical Prediction

Scaled Model Measurements

Courtesy of MIT Lincoln Laboratory
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Full Scale Measurements

Courtesy of MIT Lincoln Laboratory
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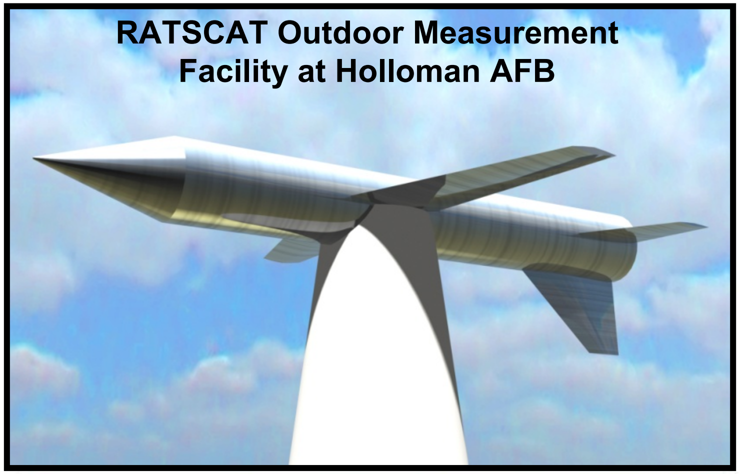
Target on Support

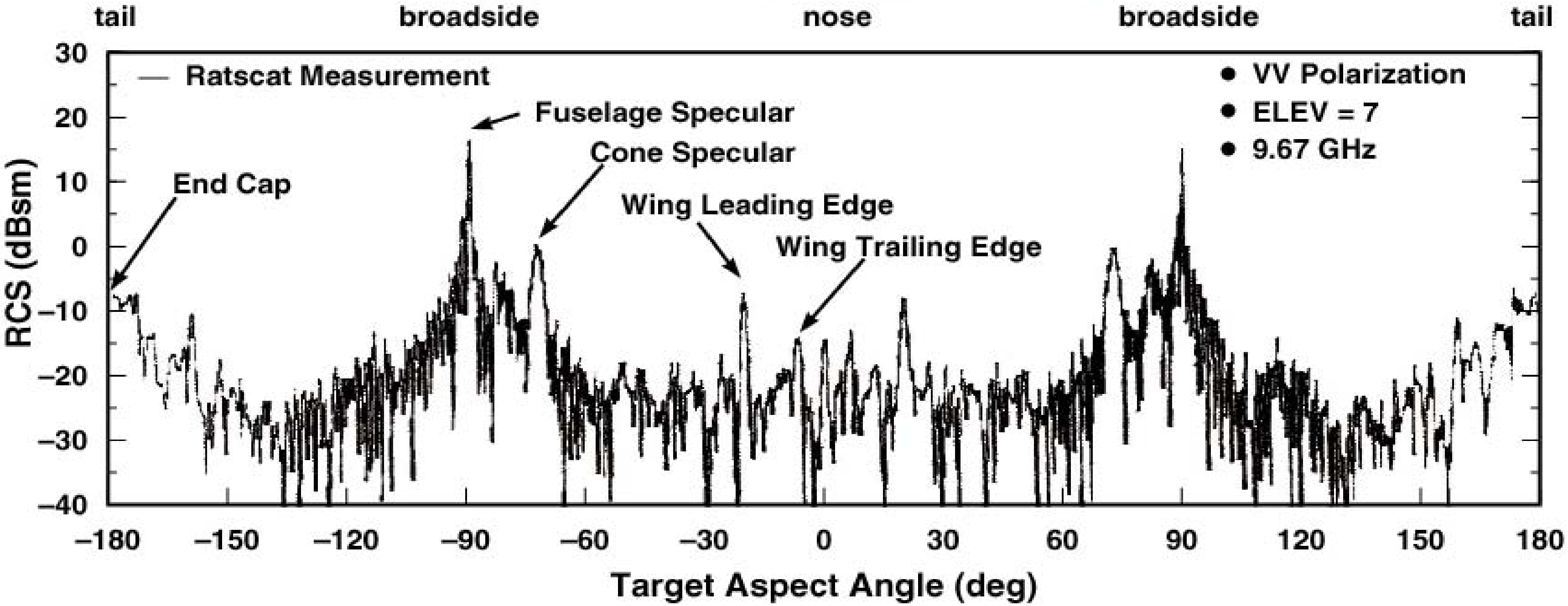
- **Foam column mounting**
  - **Dielectric properties of Styrofoam close to those of free space**
- **Metal pylon mounting**
  - **Metal pylon shaped to reduce radar reflections**
  - **Background subtraction can be used**

Derived from: http://www.af.mil/shared/media/photodb/photos/050805-F-0000S-003.jpg

Full Scale Measurement of Johnson Generic Aircraft Model (JGAM)

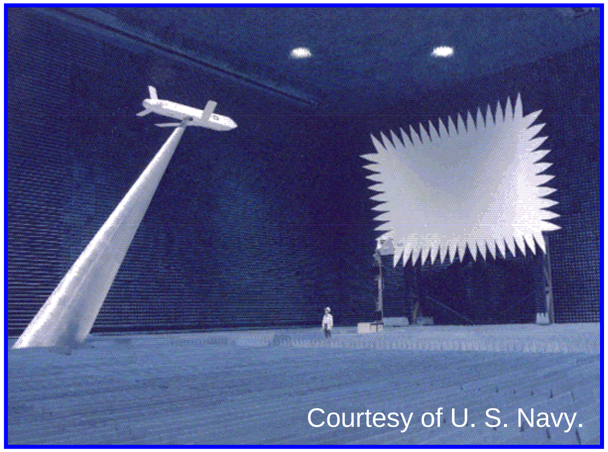
Courtesy of MIT Lincoln Laboratory
Used with Permission

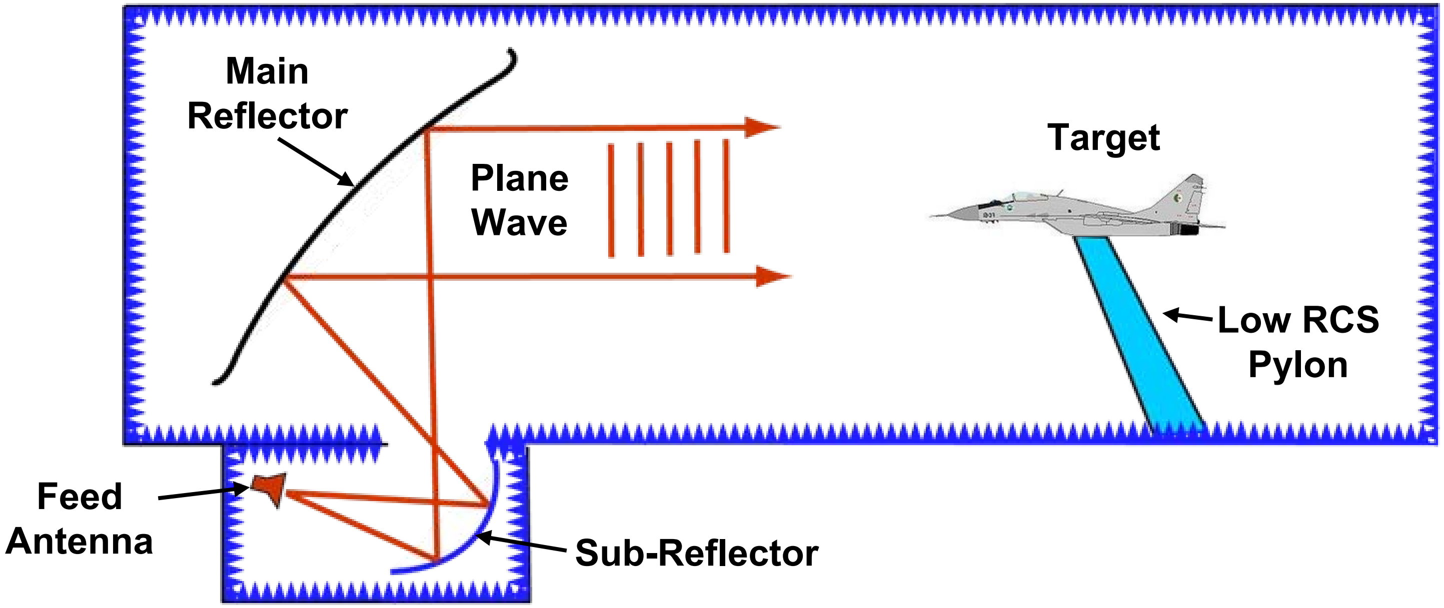




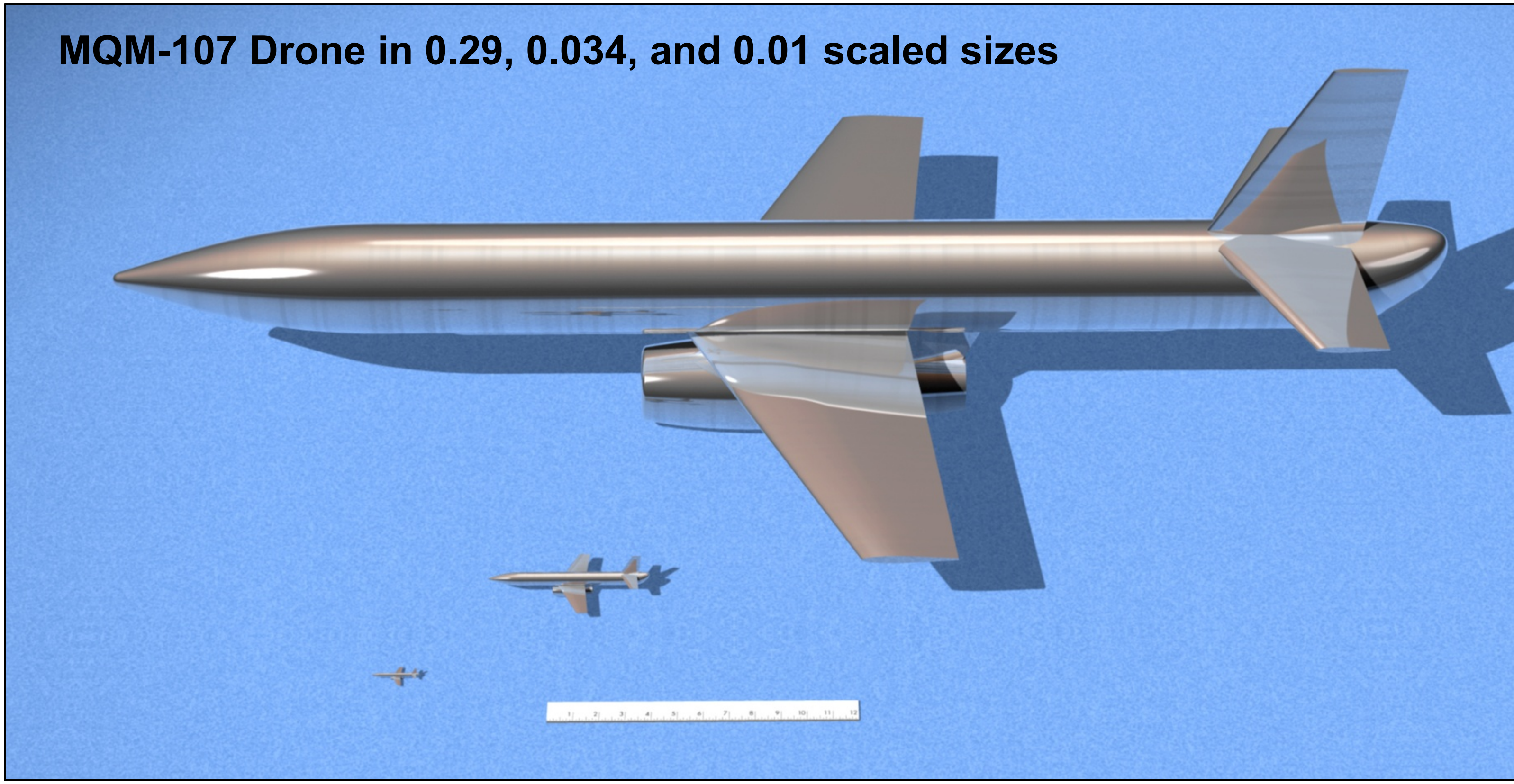
Compact Range RCS Measurement

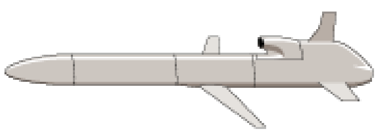
Radar Reflectivity Laboratory (Pt. Mugu) / AFRL Compact Range (WPAFB)

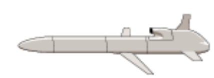




Scale Model Measurement







Full Scale
Measure at frequency f →

Scale Factor
S
(Reduced Size) →

Subscale
Measure at frequency S x F

Courtesy of MIT Lincoln Laboratory
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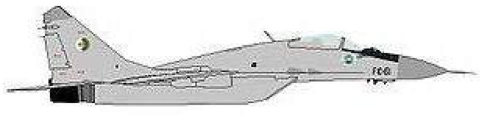
Scaling of RCS of Targets



Scale Factor
S

Quantity	Full Scale	Subscale
Length	L	L´ = L / S
Wavelength	λ	λ´ = λ / S
Frequency	f	f´ = S f
Time	t	t´ = t / S
Permittivity	ε	ε´ = ε
Permeability	μ	μ´ = μ
Conductivity	g	g´ = S g
Radar Cross Section	σ	$\sigma' = \sigma / S^2$

Outline

- **Radar cross section (RCS) of typical targets**
  - **Variation with frequency, type of target, etc.**
- **Physical scattering mechanisms and contributors to the RCS of a target**
- **Prediction of a target's radar cross section**
  - **Measurement**
  - **Theoretical Calculation**

Radar Cross Section Calculation Methods

- **Introduction**
  - A look at the few simple problems
- **RCS prediction**
  - Exact Techniques
    - Finite Difference- Time Domain Technique (FD-TD)
    - Method of Moments (MOM)
  - Approximate Techniques
    - Geometrical Optics (GO)
    - Physical Optics (PO)
    - Geometrical Theory of Diffraction (GTD)
    - Physical Theory of Diffraction (PTD)
- **Comparison of different methodologies**

Radar Cross Section of Sphere

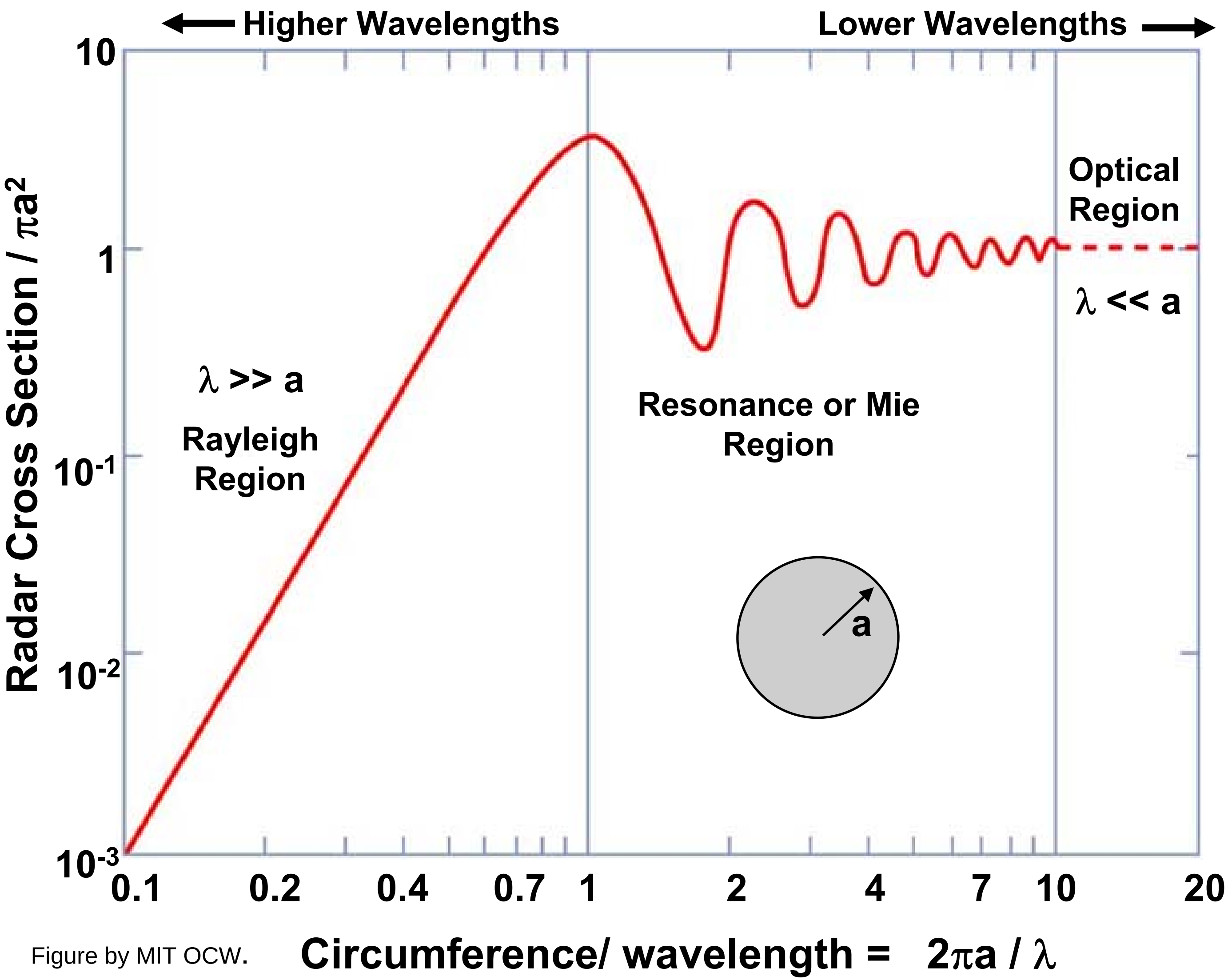


Figure by MIT OCW.

Rayleigh Region

$\lambda >> a$

$\sigma = k / \lambda^4$

Mie or Resonance Region

Oscillations

Backscattered wave interferes with creeping wave

Optical Region

$\lambda << a$

$\sigma = \pi a^2$

Surface and edge scattering occur

Radar Cross Section Calculation Issues

- Three regions of wavelength

Rayleigh	(λ >> a)
Mie / Resonance	(λ ~ a)
Optical	(λ << a)

- Other simple shapes
  - Examples: Cylinders, Flat Plates, Rods, Cones, Ogives
  - Some amenable to relatively straightforward solutions in some wavelength regions
- Complex targets:
  - Examples: Aircraft, Missiles, Ships)
  - RCS changes significantly with very small changes in frequency and / or viewing angle

    See Ref. 6 (Levanon), problem 2-1 or Ref. 2 (Skolnik) page 57
- We will spend the rest of the lecture studying the different basic methods of calculating radar cross sections

High Frequency RCS Approximations
(Simple Scattering Features)

Scattering Feature	Orientation	Approximate RCS
Corner Reflector	Axis of symmetry along LOS	$4\pi A_{eff}^2 / \lambda^2$
Flat Plate	Surface perpendicular to LOS	$4\pi A^2 / \lambda^2$
Singly Curved Surface	Surface perpendicular to LOS	$4\pi A^2 / \lambda^2$
Doubly Curved Surface	Surface perpendicular to LOS	$\pi a_1 a_2$
Straight Edge	Edge perpendicular to LOS	λ^2 / π
Curved Edge	Edge element perpendicular to LOS	$a \lambda / 2$
Cone Tip	Axial incidence	$\lambda^2 \sin^4(\alpha / 2)$

Where:
LOS = line of sight
A_{eff} = effective area contributing to multiple internal reflections
A = actual area of plate
a = mean radius of curvature; L = length of slanted surface
a_1 and a_2 = principal radii of surface curvature in orthogonal planes
L = edge length
a = radius of edge contour
α = half angle of the cone

Adapted from Knott is Skolnik Reference 3

Radar Cross Section Calculation Issues

- Three regions of wavelength

Rayleigh	($\lambda >> a$)
Mie / Resonance	($\lambda \sim a$)
Optical	($\lambda << a$)

- Other simple shapes
  - Examples: Cylinders, Flat Plates, Rods, Cones, Ogives
  - <u>Some</u> amenable to relatively straightforward solutions in <u>some</u> wavelength regions
- Complex targets:
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RCS Calculation - Overview

- Electromagnetism Problem
  - A plane wave with electric field, $\vec{E}_I$, impinges on the target of interest and some of the energy scatters back to the radar antenna
  - Since, the radar cross section is given by: $\sigma = \lim_{r \to \infty} 4 \pi r^2 \frac{\left|\vec{E}_S\right|^2}{\left|\vec{E}_I\right|^2}$
  - All we need to do is use Maxwell's Equations to calculate the scattered electric field $\vec{E}_S$
  - That's easier said that done
  - Before we examine in detail these different techniques, let's review briefly the necessary electromagnetism concepts and formulae, in the next few viewgraphs

Maxwell's Equations

- Source free region of space:

$$\vec{\nabla} \times \vec{E}(\vec{r},t) = -\frac{\partial \vec{B}(\vec{r},t)}{\partial t}$$

$$\vec{\nabla} \times \vec{H}(\vec{r},t) = \frac{\partial \vec{D}(\vec{r},t)}{\partial t}$$

$$\nabla \cdot \vec{D}(\vec{r},t) = 0$$

$$\nabla \cdot \vec{B}(\vec{r},t) = 0$$

- Free space constitutive relations:

$$\vec{D}(\vec{r},t) = \varepsilon_o \vec{E}(\vec{r},t)$$

$$\vec{B}(\vec{r},t) = \mu_o \vec{H}(\vec{r},t)$$

ε_o = Free space permittivity

μ_o = Free space permeability

Maxwell's Equations in Time-Harmonic Form

- **Source free region:**

$$\vec{\nabla} \times \vec{E}(\vec{r}) = i\omega \vec{B}(\vec{r})$$

$$\vec{\nabla} \times \vec{H}(\vec{r}) = -i\omega \vec{D}(\vec{r})$$

$$\nabla \cdot \vec{D}(\vec{r}) = 0$$

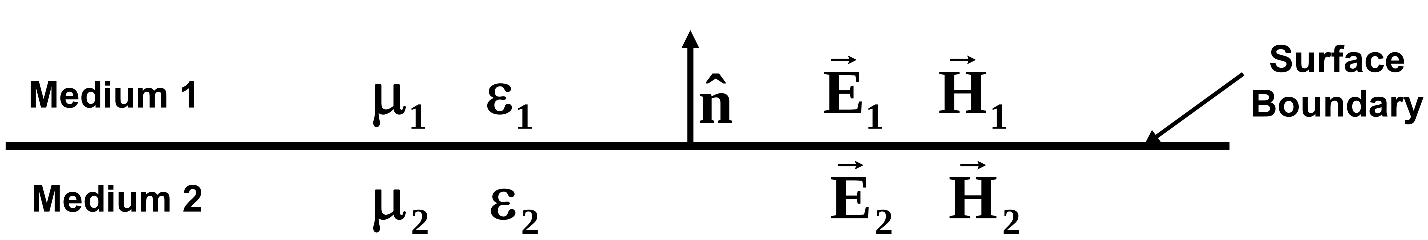
$$\nabla \cdot \vec{B}(\vec{r}) = 0$$

- **Time dependence**

$$\vec{E}(\vec{r},t) = \mathrm{Re}\left\{\vec{E}(\vec{r})e^{-i\omega t}\right\}$$

$$\vec{H}(\vec{r},t) = \mathrm{Re}\left\{\vec{H}(\vec{r})e^{-i\omega t}\right\}$$

Boundary Conditions



- Tangential components of $\vec{E}$ and $\vec{H}$ are continuous:

$$\hat{n} \times \vec{E}_1 = \hat{n} \times \vec{E}_2$$

$$\hat{n} \times \vec{H}_1 = \hat{n} \times \vec{H}_2$$

- For surfaces that are perfect conductors:

$$\hat{n} \times \vec{E} = 0$$

- Radiation condition:
  - As $r \rightarrow \infty$ $\quad \vec{E}(\vec{r}) \propto \frac{1}{r}$

Scattering Matrix

- For a linear polarization basis

Scattering Matrix - S

$$\vec{E}_S = \begin{bmatrix} E_{VS} \\ E_{HS} \end{bmatrix} = \frac{e^{ikr}}{r} \begin{bmatrix} S_{VV} & S_{VH} \\ S_{HV} & S_{HH} \end{bmatrix} \begin{bmatrix} E_{VI} \\ E_{HI} \end{bmatrix}$$

- The incident field polarization is related to the scattered field polarization by this Scattering Matrix - S

$$\sigma_{VV} = 4\pi \left| S_{VV} \right|^2$$

$$\sigma_{HH} = 4\pi \left| S_{HH} \right|^2$$

$$\sigma_{VH} = 4\pi \left| S_{VH} \right|^2$$

- For and a reciprocal medium and for monostatic radar cross section:

$$\sigma_{RR},\ \sigma_{LL}, \sigma_{RL}$$

- For a circular polarization basis

$$\sigma_{VH} = \sigma_{HV}$$

Radar Cross Section Calculation Methods

- Introduction
  - A look at the few simple problems
- RCS prediction
  - Exact Techniques
    - Finite Difference- Time Domain Technique (FD-TD)
    - Method of Moments (MOM)
  - Approximate Techniques
    - Geometrical Optics (GO)
    - Physical Optics (PO)
    - Geometrical Theory of Diffraction (GTD)
    - Physical Theory of Diffraction (PTD)
- Comparison of different methodologies

Methods of Radar Cross Section Calculation

RCS Method	Approach to Determine Surface Currents
Finite Difference-Time Domain (FD-TD)	Solve Differential Form of Maxwell's Equation's for Exact Fields
Method of Moments (MoM)	Solve Integral Form of Maxwell's Equation's for Exact Currents
Geometrical Optics (GO)	Current Contribution Assumed to Vanish Except at Isolated Specular Points
Physical Optics (PO)	Currents Approximated by Tangent Plane Method
Geometrical Theory of Diffraction (GTD)	Geometrical Optics with Added Edge Current Contribution
Physical Theory of Diffraction (PTD)	Physical Optics with Added Edge Current Contribution

Finite Difference- Time Domain (FD-TD) Overview

- Exact method for calculation radar cross section
- Solve differential form of Maxwell's equations
  - The change in the E field, in time, is dependent on the change in the H field, across space, and visa versa
- The differential equations are transformed to difference equations
  - These difference equations are used to sequentially calculate the E field at one time and the use those E field calculations to calculate H field at an incrementally greater time; etc. etc.
    - Called "Marching in Time"
- These time stepped E and H field calculations avoid the necessity of solving simultaneous equations
- Good approach for structures with varying electric and magnetic properties and for cavities

Maxwell's Equations in Rectangular Coordinates

- Examine 2 D problem – no y dependence: $\frac{\partial}{\partial y} = 0$

- Equations decouple into H-field polarization and E-field polarization

$$\frac{\partial}{\partial y} H_Z - \frac{\partial}{\partial z} H_Y = \varepsilon_o \frac{\partial}{\partial t} E_X \qquad \frac{\partial}{\partial y} E_Z - \frac{\partial}{\partial z} E_Y = -\mu_o \frac{\partial}{\partial t} H_X$$

$$\frac{\partial}{\partial z} E_X - \frac{\partial}{\partial x} E_Z = -\mu_o \frac{\partial}{\partial t} H_Y \qquad \frac{\partial}{\partial z} H_X - \frac{\partial}{\partial x} H_Z = \varepsilon_o \frac{\partial}{\partial t} E_Y$$

$$\frac{\partial}{\partial x} H_Y - \frac{\partial}{\partial y} H_X = \varepsilon_o \frac{\partial}{\partial t} E_Z \qquad \frac{\partial}{\partial x} E_Y - \frac{\partial}{\partial y} E_X = -\mu_o \frac{\partial}{\partial t} H_Z$$

- H-field polarization

  $H_Y \quad E_X \quad E_Z$

- E-field polarization

  $E_Y \quad H_X \quad H_Z$

Maxwell's Equations in Rectangular Coordinates

- Examine 2 D problem – no y dependence: $\frac{\partial}{\partial y} = 0$
- Equations decouple into H-field polarization and E-field polarization

$$\frac{\partial}{\partial y} H_Z - \frac{\partial}{\partial z} H_Y = \varepsilon_o \frac{\partial}{\partial t} E_X \qquad \left(\frac{\partial}{\partial y} H_Z = 0\right)$$

$$\frac{\partial}{\partial z} E_X - \frac{\partial}{\partial x} E_Z = -\mu_o \frac{\partial}{\partial t} H_Y$$

$$\frac{\partial}{\partial x} H_Y - \frac{\partial}{\partial y} H_X = \varepsilon_o \frac{\partial}{\partial t} E_Z \qquad \left(\frac{\partial}{\partial y} H_X = 0\right)$$

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$$\frac{\partial}{\partial x} E_Y - \frac{\partial}{\partial y} E_X = -\mu_o \frac{\partial}{\partial t} H_Z \qquad \left(\frac{\partial}{\partial y} E_X = 0\right)$$

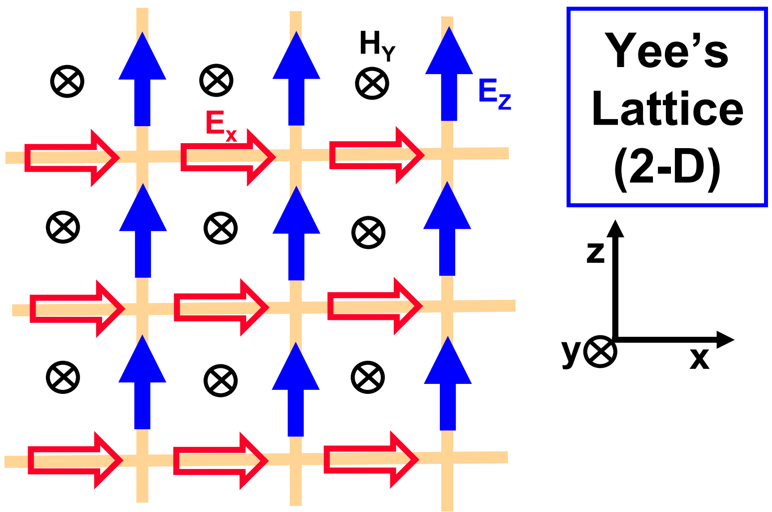
- H-field polarization
  $H_Y \quad E_X \quad E_Z$

- E-field polarization
  $E_Y \quad H_X \quad H_Z$

Discrete Form of Maxwell's Equations

- **H-field polarization:**

$$-\mu_o \frac{\partial}{\partial t} H_Y(x,y,t) = \frac{\partial}{\partial z} E_X(x,y,t) - \frac{\partial}{\partial x} E_Z(x,y,t)$$



- **Discrete form:**

$$-\frac{\mu_o}{\Delta_T}\left[H_Y\left(x_o+\frac{\Delta_X}{2}, z_o+\frac{\Delta_Z}{2}, t_o+\frac{\Delta_T}{2}\right) - H_Y\left(x_o+\frac{\Delta_X}{2}, z_o+\frac{\Delta_Z}{2}, t_o-\frac{\Delta_T}{2}\right)\right]$$

$$=\frac{1}{\Delta_Z}\left[E_X\left(x_o+\frac{\Delta_X}{2}, z_o+\Delta_Z, t_o\right) - E_X\left(x_o+\frac{\Delta_X}{2}, z_o, t_o\right)\right]$$

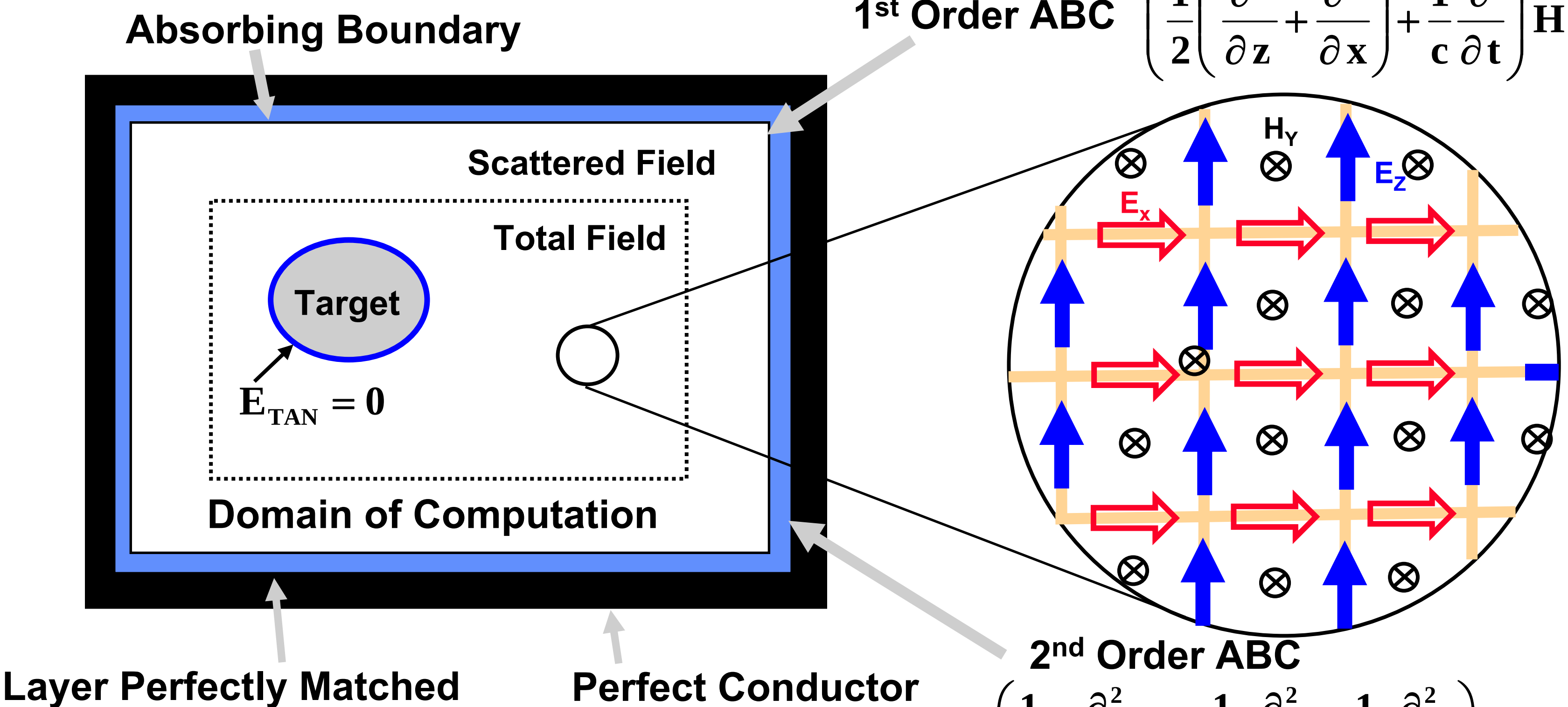
$$-\frac{1}{\Delta_X}\left[E_Z\left(x_o+\Delta_X, z_o+\frac{\Delta_Z}{2}, t_o\right) - E_Z\left(x_o, z_o+\frac{\Delta_Z}{2}, t_o\right)\right]$$

- **Electric and magnetic fields are calculated alternately by the marching in time method**

FD-TD Calculations and Absorbing Boundary Conditions (ABC)

1st Order ABC

$$\left(\frac{1}{2}\left(\frac{\partial}{\partial z}+\frac{\partial}{\partial x}\right)+\frac{1}{c}\frac{\partial}{\partial t}\right)H_y=0$$



2nd Order ABC

$$\left(\frac{1}{c}\frac{\partial^2}{\partial x\,\partial t}+\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\frac{1}{2}\frac{\partial^2}{\partial x^2}\right)H_y=0$$

- Absorbing Boundary Condition (ABC) Used to Limit Computational Domain
  - Reflections at exterior boundary are minimized
  - Traditional ABC's model field as outgoing wave to estimate field quantities outside domain
  - More recent perfectly matched layer (PML) model uses non-physical layer, that absorbs waves

RCS Calculations Using the FD-TD Method

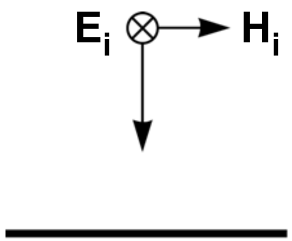
- **Single frequency RCS calculations**
  - Excite with sinusoidal incident wave
  - Run computation until steady state is reached
  - Calculate amplitude and phase of scattered wave
- **Multiple frequency RCS calculations**
  - Excite with Gaussian pulse incident wave
  - Calculate transient response
  - Take Fourier transform of incident pulse and transient response
  - Calculate ratios of these transforms to obtain RCS at multiple frequencies

From Atkins, Reference 5
Courtesy of MIT Lincoln Laboratory

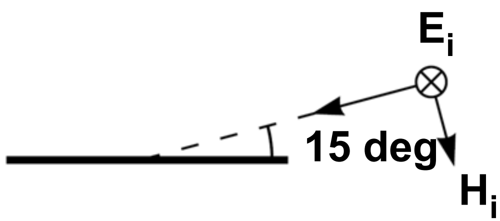
Description of Scattering Cases on Video

Finite Difference Time Domain (FDTD) Simulations

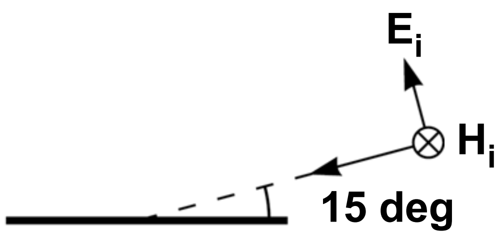
Case 1 – Plate I



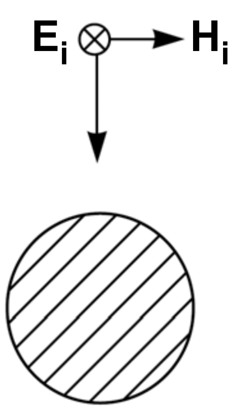
Case 2 – Plate II



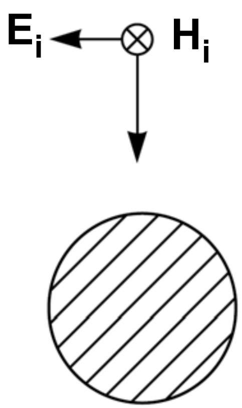
Case 3 – Plate III



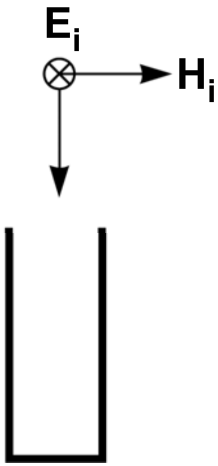
Case 4 – Cylinder I



Case 5 – Cylinder II



Case 6 – Cavity



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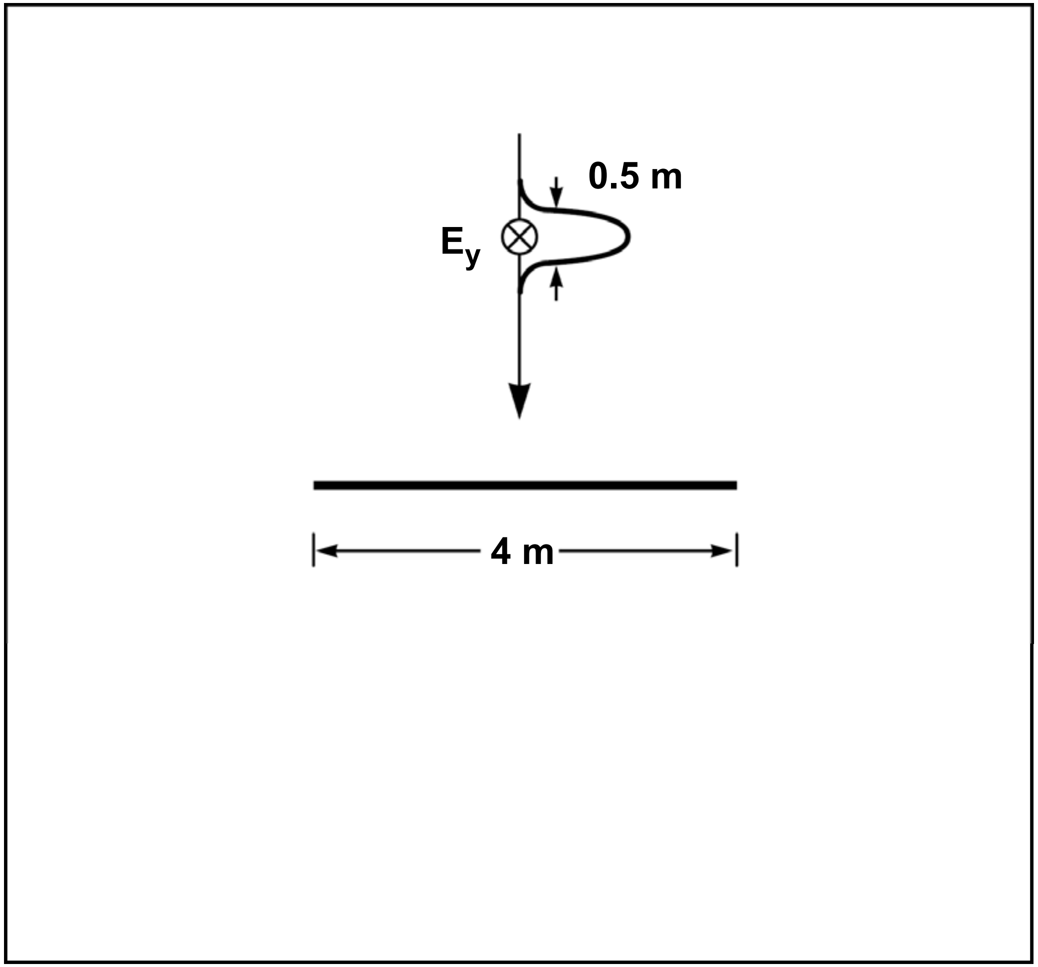
FD-TD Simulation of Scattering by Strip

Case 1

- Gaussian pulse plane wave incidence
- E-field polarization (E_y plotted)
- Phenomena: specular reflection



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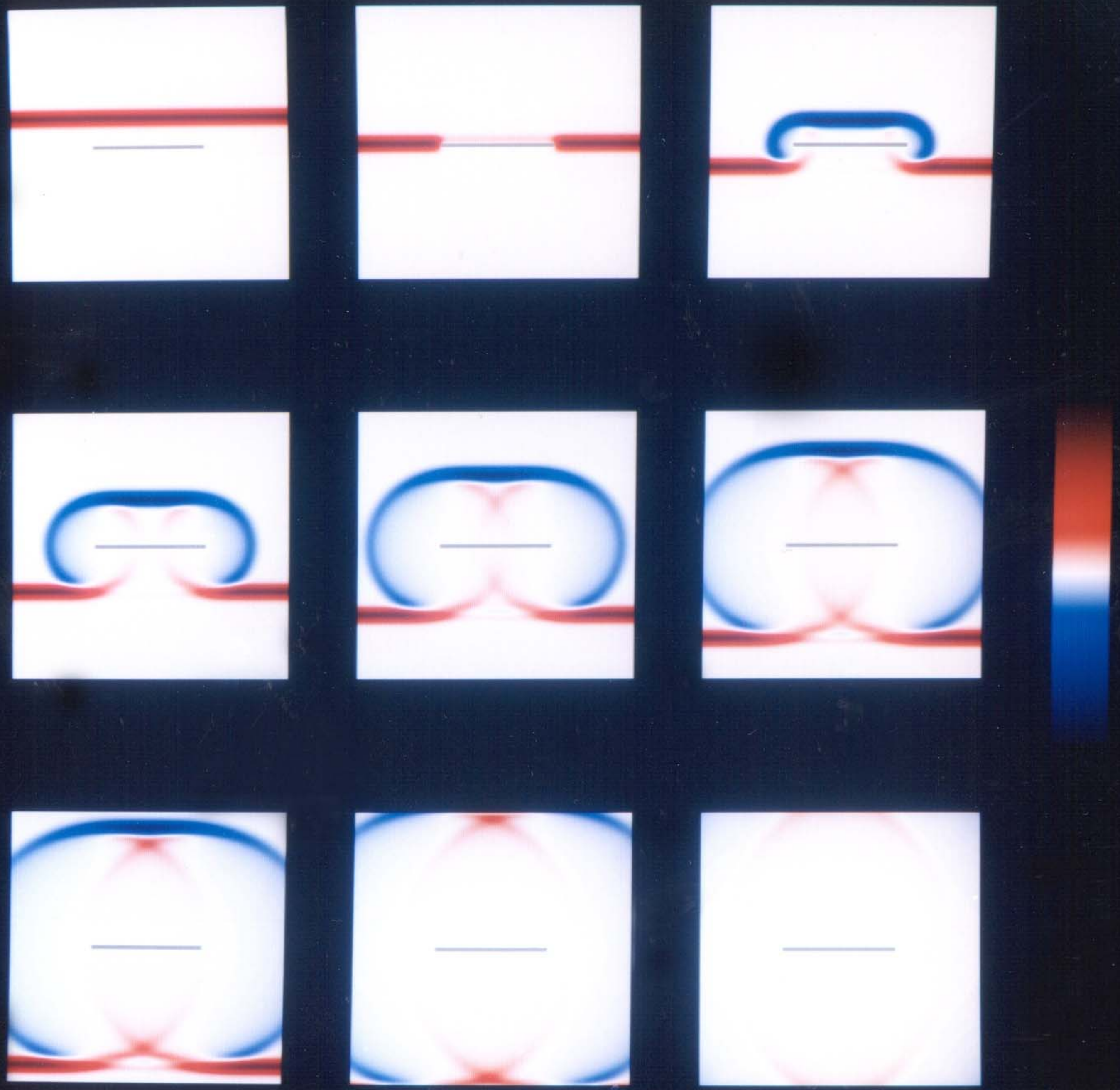
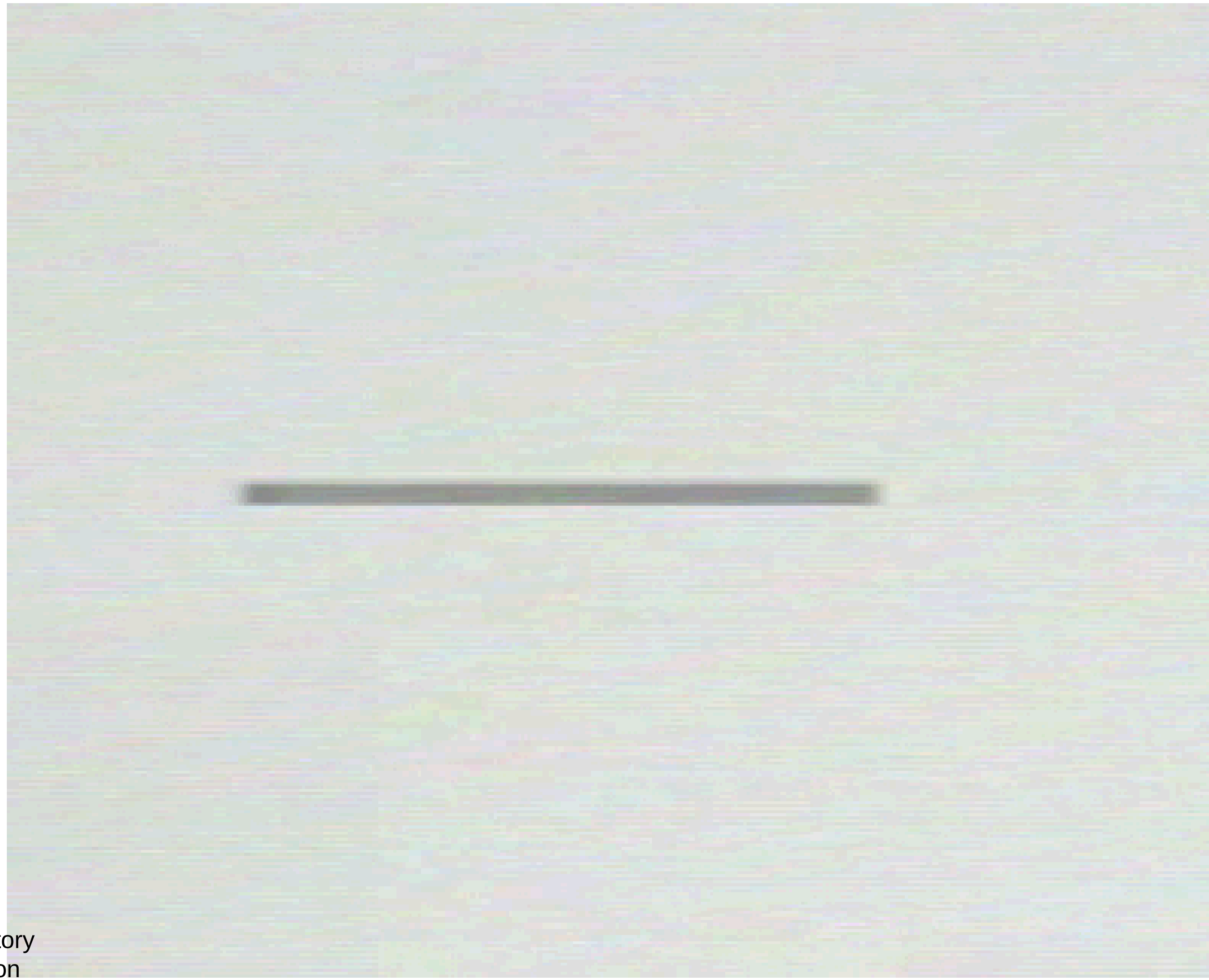
Case 1



FD-TD Simulation of Scattering by Strip

Case 1



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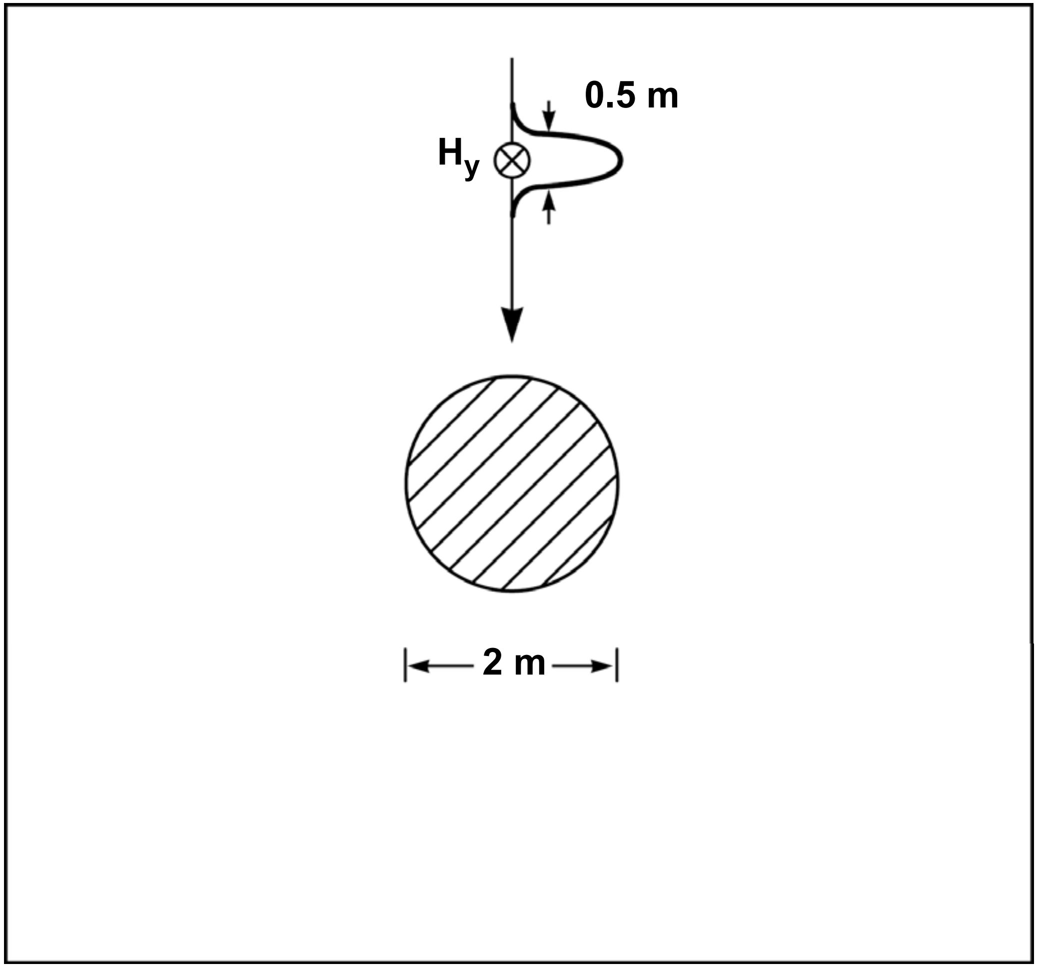
FD-TD Simulation of Scattering by Cylinder

Case 5

- **Gaussian pulse plane wave incidence**
- **H-field polarization (H_y plotted)**
- **Phenomena: creeping wave**



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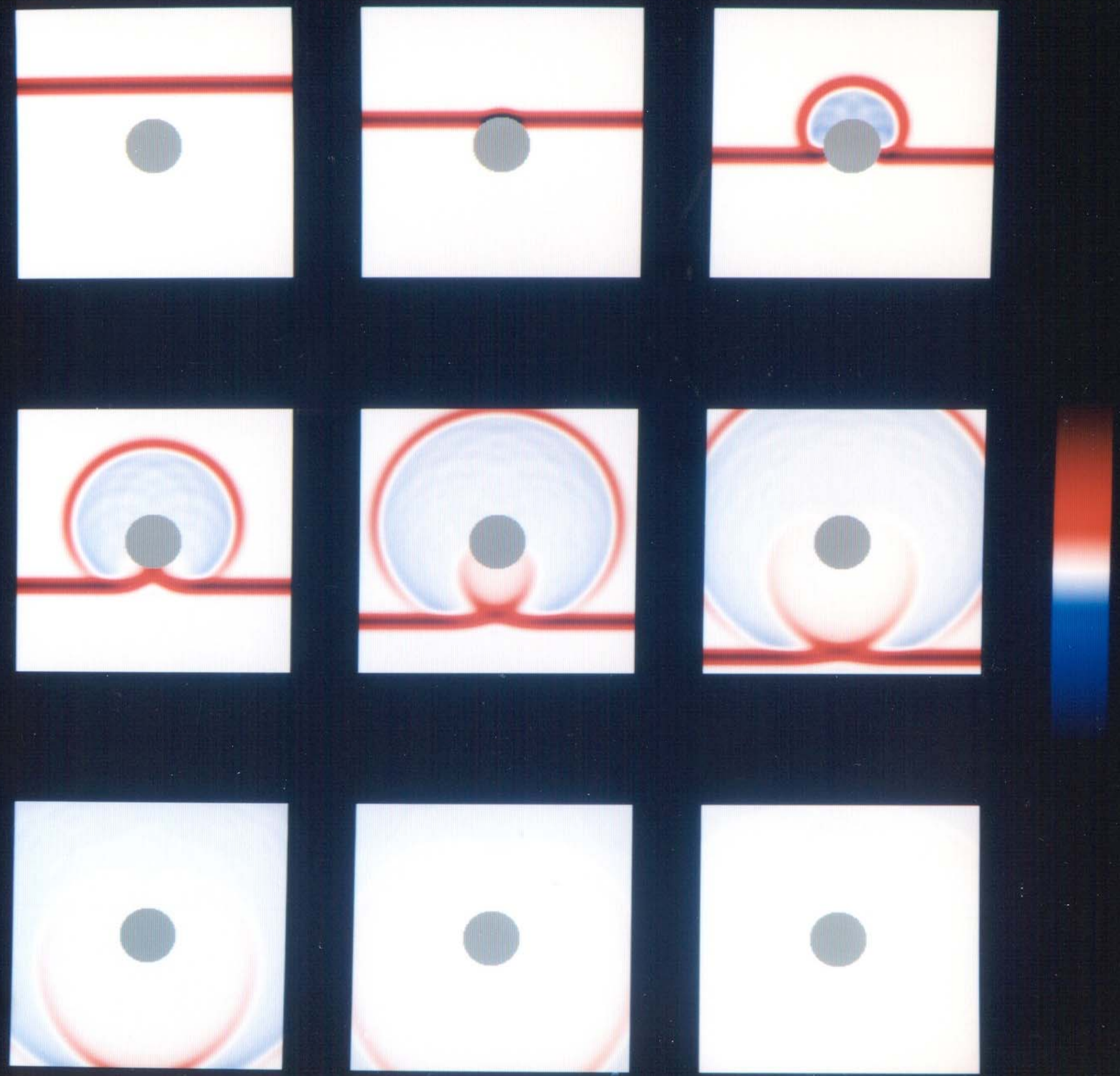
Case 5



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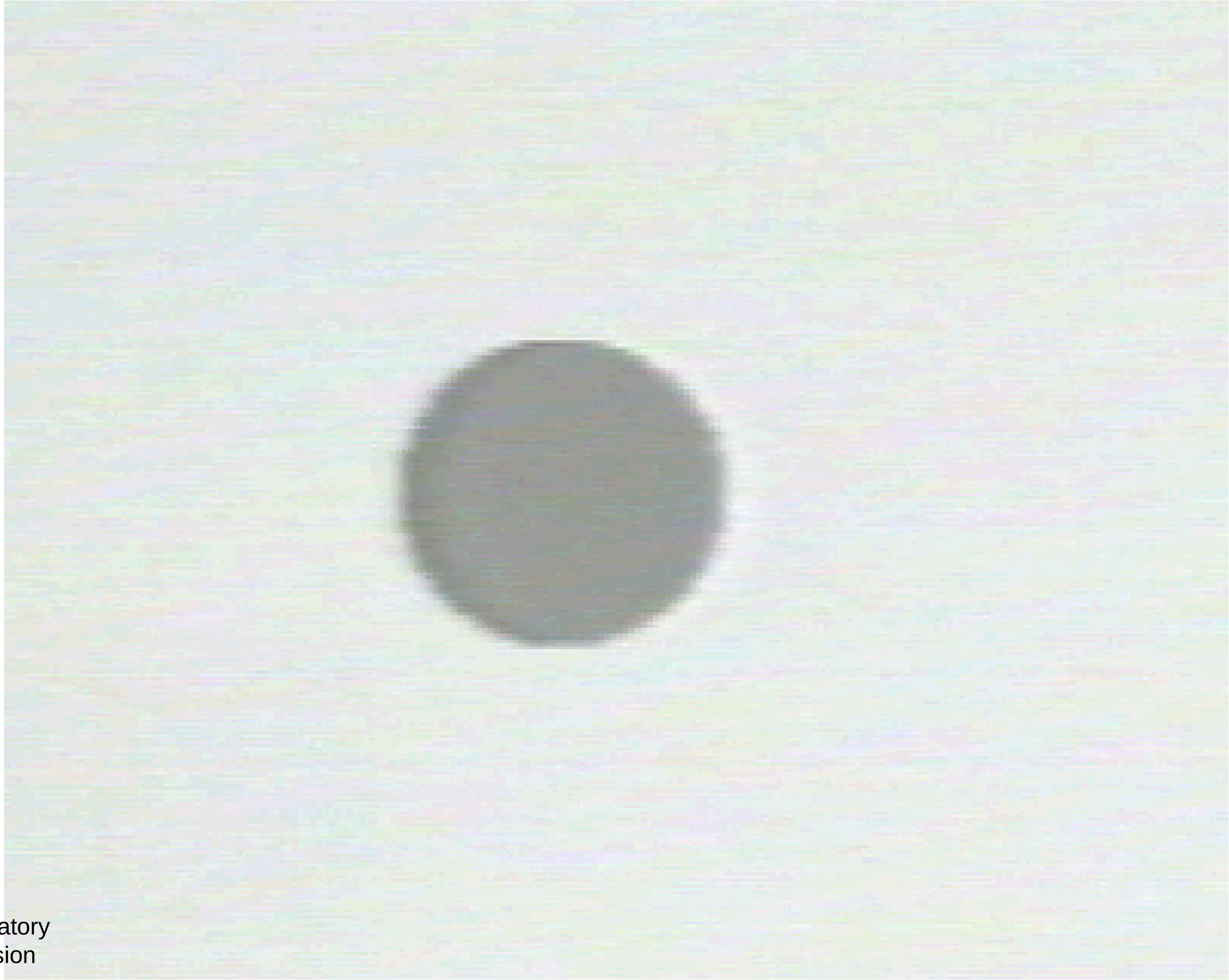
FD-TD Simulation of Scattering by Cylinder

Case 5



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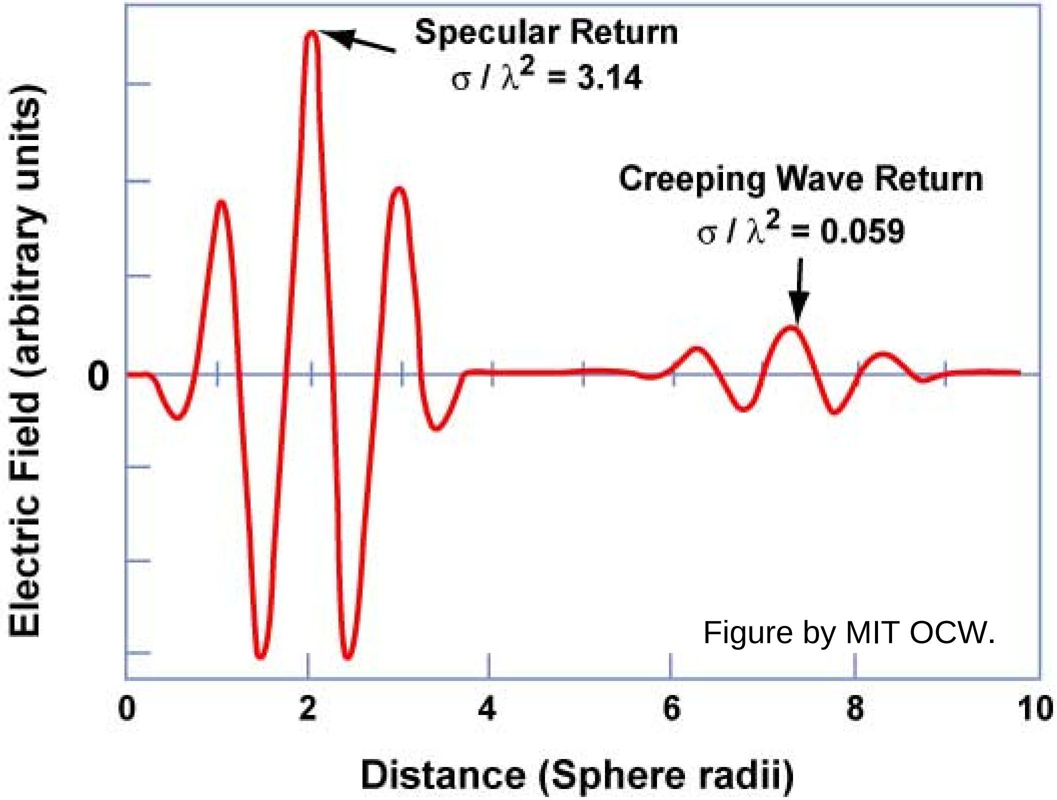
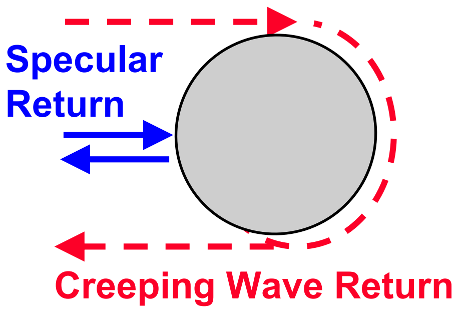
Backscatter of Short Pulse from Sphere





Radius of sphere is equal to the radar wavelength