Chapter VII MoM VIE Approach to a Metal-Dielectric Antenna

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7.1. MoM equations for a metal-dielectric structure

a. Scattering problem

The present derivation follows the derivation given in [1] for the VIE approach to the metal-dielectric antennas. The complete moment equations essentially combine the results of Chapters V and VI together. The new feature is a (symmetric) interaction part of the total impedance matrix, which describes metal-to-dielectric (or dielectric-to-metal) interaction. Similar to Chapters V and VI, the scattering problem is considered. The total electric field (scattering problem) is a combination of the incident field (labeled by superscript $i$) and the scattered field (labeled by superscript $s$), i.e.

$$\vec{E} = \vec{E}^i + \vec{E}^s$$

(7.1)

Let $V$ (bounded by surface $\Omega$) denote the volume of a lossy, inhomogeneous, dielectric body with (complex) dielectric constant $\varepsilon(\vec{r}) = \varepsilon(\vec{r}) - j\sigma(\vec{r})/\omega$, where $\varepsilon$ and $\sigma$ are the medium permittivity and conductivity when $\vec{r}$ is in $V$. Let a metal surface $S$ be attached to this dielectric object or be in the vicinity of it.

The incident field is the incoming signal for the scattering problem. The scattered electric field $\vec{E}^s$ in this case will have two components. One is due to volume polarization currents in the dielectric volume $V$ and associated bound charges on the boundaries of an inhomogeneous dielectric region, and the other component is due to surface conduction currents and free charges on the metal surface $S$. Using the expressions for the scattered field in terms of the electric and magnetic potentials $\vec{A}$ and $\Phi$, one has

$$\vec{E}^s = -j\omega\vec{A}(\vec{r}) - \nabla\Phi(\vec{r}) - j\omega A_M(\vec{r}) - \nabla\Phi_M(\vec{r}) \quad \vec{r} \text{ in } V$$

(7.2)

$$\vec{E}^s = -j\omega\vec{A}_M(\vec{r}) - \nabla\Phi_M(\vec{r}) - j\omega\vec{A}(\vec{r}) - \nabla\Phi(\vec{r}) \quad \vec{r} \text{ on } S$$

(7.3)
where index $M$ refers to the metal surface $S$. The magnetic vector potential $\vec{A}(\vec{r})$ and electric potential $\Phi(\vec{r})$ carry their usual meanings corresponding to metal and dielectric [2]. Since

$$\vec{D} = \varepsilon\vec{E} \quad \text{in the dielectric volume } V$$

(7.4)

$$\vec{E}_{\tan} = 0 \quad \text{on the metal surface } S$$

(7.5)

using the expressions for $\vec{E}$ and $\vec{E}^s$, we can write the EFIE as

$$\vec{E}^i = \frac{\vec{D}(\vec{r})}{\varepsilon(\vec{r})} + j\omega\vec{A}(\vec{r}) + \nabla\Phi(\vec{r}) + j\omega\vec{A}_M(\vec{r}) + \nabla\Phi_M(\vec{r}) \quad \vec{r} \ \text{in } V$$

(7.6)

$$\vec{E}_{\tan}^i = \left[ + j\omega\vec{A}_M(\vec{r}) + \nabla\Phi_M(\vec{r}) + j\omega\vec{A}(\vec{r}) + \nabla\Phi(\vec{r}) \right]_{\text{on } S}$$

(7.7)

**b. Test functions**

Assume that some test functions, $K(\vec{r})\vec{f}_m(\vec{r})$, $m = 1 \ldots N_D$, cover the entire dielectric volume $V$. Multiplication of equation (7.6) by $K(\vec{r})\vec{f}_m(\vec{r})$ and integration over volume $V$ gives $N_D$ equations

$$\int_V K(\vec{r})\vec{f}_m(\vec{r}) \cdot \vec{E}^i d\vec{r} =$$

(7.8)
Simplifying the last volume integral by applying Stokes theorem for every individual tetrahedron in the manner similar to the simplification of the last volume integral in Eq. (6.9) of Chapter VI yields

$$\int_{\Omega} K(\vec{r}) \vec{f}_m(\vec{r}) \cdot \nabla \Phi_M \, dv = -\int_{\Omega} K(\vec{r}) \Phi_M \left( \nabla \cdot \vec{f}_m(\vec{r}) \right) \, dv + \int_{\Omega} K(\vec{r}) \Phi_M \left( \hat{n}(\vec{r}) \cdot \vec{f}_m(\vec{r}) \right) \, ds$$

However the volume basis functions are divergenceless. Hence

$$\int_{\Omega} K(\vec{r}) \vec{f}_m(\vec{r}) \cdot \nabla \Phi \, dv = \int_{\Omega} K(\vec{r}) \Phi_S \left( \hat{n}(\vec{r}) \cdot \vec{f}_m(\vec{r}) \right) \, ds = \int_{\Omega} K(\vec{r}) \Phi_S \cdot f_{\perp mq}(\vec{r}) \, ds \tag{7.9}$$

where \( \hat{n} \) is the unit outer normal to the surface \( \Omega \) and \( f_{\perp mq}(\vec{r}) \) is the outer normal component of the basis function \( \vec{f}_m(\vec{r}) \) on face \( q \). Substituting the values from equations (7.5) and (7.9) in equation (7.8) gives

$$\int_{\Omega} K(\vec{r}) \vec{f}_m(\vec{r}) \cdot \vec{E}^i \, dv = \left[ \int_{\Omega} K(\vec{r}) \vec{f}_m(\vec{r}) \cdot \frac{\vec{D}(\vec{r})}{\vec{E}(\vec{r})} \, dv + j\omega \int_{\Omega} K(\vec{r}) \vec{f}_m(\vec{r}) \cdot \vec{A} \, dv + \int_{\Omega} \hat{K}_q \Phi f_{\perp mq}(\vec{r}) \, ds \right] \underbrace{Z_{\text{DD}}}_{\text{DD}} + j\omega \int_{\Omega} K(\vec{r}) \vec{f}_m(\vec{r}) \cdot \vec{A} \, dv + \int_{\Omega} \hat{K}_q \Phi f_{\perp mq}(\vec{r}) \, ds \right] \underbrace{Z_{\text{DM}}}_{\text{DM}} \tag{7.10}$$

The process of converting the contrast, \( K(\vec{r}) \), to the differential contrast, \( \hat{K} \), is exactly the same as explained in Chapter VI. The term on the right-hand side of equation (2.3.10), labeled \( Z_{\text{DD}} \), is exactly the right-hand side of equation (6.9) from Chapter VI for the pure dielectric. The term, labeled \( Z_{\text{DM}} \), describes the contribution of radiation from the metal surface to the dielectric volume.

Now assume that the surface test functions, \( \vec{f}_m^M(\vec{r}) \), \( m = 1 \ldots N_M \), cover the entire metal surface \( S \) and do not have a component normal to the surface. Multiplication of equation (7.7) by \( \vec{f}_m^M(\vec{r}) \) and integration over surface \( S \) gives \( N_M \) equations.
since according to Stoke’s theorem,

\[
\int_{S} \vec{f}_{m}^{S}(\vec{r}) \cdot \vec{E}^{i} d\vec{s} = -\int_{S} \Phi \left( \nabla \cdot \vec{f}_{m}^{S}(\vec{r}) \right) d\vec{s}
\]  

(7.12)

The term on the right-hand side of equation (7.11), labeled $Z_{MM}^{MM}$, is exactly the right-hand side of equation (5.6) from Chapter V for the pure metal. The term, labeled $Z_{MD}^{MD}$, describes the contribution of radiation from the dielectric volume to the metal surface.

c. Source functions and moment equations

The material in this Section is essentially the combination of Chapters V and VI where the MoM equations are obtained for the pure metal and pure dielectric structure separately. The moment equations are obtained if we substitute expansions for potentials in terms of the corresponding source basis functions into equations (7.10), (7.11). In terms of symbolic notations,

\[
\sum_{n=1}^{N_{D}} \hat{Z}_{mn}^{DD} D_{n} + \sum_{n=1}^{N_{M}} \hat{Z}_{mn}^{DM} I_{n} = \nu_{m}^{D} \quad m = 1, \ldots N_{D}
\]  

(7.13)

\[
\sum_{n=1}^{N_{M}} \hat{Z}_{mn}^{MM} I_{n} + \sum_{n=1}^{N_{D}} \hat{Z}_{mn}^{MD} D_{n} = \nu_{m}^{M} \quad m = 1, \ldots N_{M}
\]  

(7.14)

where

\[
\nu_{m}^{D} = \int_{V} K(\vec{r}) \vec{f}_{m}(\vec{r}) \cdot \vec{E}^{i} d\vec{v}, \quad \nu_{m}^{M} = \int_{S} \vec{f}_{m}^{M}(\vec{r}) \cdot \vec{E}^{i} d\vec{s}
\]  

(7.15)
The square impedance matrices $\hat{Z}^{MM}$ and $\hat{Z}^{DD}$ have been described in Chapters V and VI, respectively. They will not be repeated here. The new part, however, are the mutual rectangular impedance matrixes $\hat{Z}^{MD}$ and $\hat{Z}^{DM}$. One has

$$Z_{mn}^{MD} = -\frac{\omega^2 \mu_0}{4\pi} \sum_{p=1}^{2} \sum_{p'=1}^{2} K_{p}^{*} \int \int \hat{f}_{n}^{M}(\vec{r}) \cdot \hat{f}_{m}^{p}(\vec{r}') g(|\vec{r} - \vec{r}'|) d\vec{r}' ds$$

$$- \frac{1}{4\pi \epsilon_0} \sum_{i=1}^{Q} \hat{K}_{q}^{*} \int \int (\nabla_{S} \cdot \hat{f}_{n}^{M}(\vec{r})) \hat{f}_{mq}^{\perp}(\vec{r}') g(|\vec{r} - \vec{r}'|) d\Omega' ds$$

$$m = 1, \ldots, N_{D}; \ n = 1, \ldots, N_{M} \tag{7.16}$$

$$Z_{mn}^{DM} = \frac{j \omega \mu_0}{4\pi} \sum_{p=1}^{2} \sum_{p'=1}^{2} K_{p}^{*} \int \int \hat{f}_{m}^{p}(\vec{r}) \cdot \hat{f}_{n}^{S}(\vec{r}') g(|\vec{r} - \vec{r}'|) ds' d\vec{r}$$

$$+ \frac{j \omega}{4\pi \epsilon_0 \omega} \sum_{i=1}^{Q} \hat{K}_{q}^{*} \int \int f_{mq}^{\perp}(\vec{r}) \cdot (\nabla_{S} \cdot \hat{f}_{m}^{S}(\vec{r}')) g(|\vec{r} - \vec{r}'|) ds' d\Omega$$

$$n = 1, \ldots, N_{D}; \ m = 1, \ldots, N_{M} \tag{7.17}$$

From equations (7.16) and (7.17) one can see that

$$\hat{Z}^{DM} = (\hat{Z}^{MD})^{T} / (j \omega) \tag{7.18}$$

where the superscript $T$ denotes the transpose matrix.

### 7.2. Total impedance matrix

The total impedance matrix is obtained by combining the metal impedance matrix $\hat{Z}^{MM}$, the dielectric impedance matrix $\hat{Z}^{DD}$, and the mutual impedance matrices $\hat{Z}^{DM}$ and $\hat{Z}^{MD}$ in the form
The impedance matrix \( \hat{Z} \) can be converted to a symmetric matrix form by using trivial transformations. One way of achieving it is

\[
\hat{Z} = \begin{bmatrix}
  j\omega \hat{Z}_{MM} & j\omega \hat{Z}_{MD} \\
  j\omega \hat{Z}_{DM} & j\omega \hat{Z}_{DD}
\end{bmatrix}
\]  

(7.20)

Once the matrix \( \hat{Z} \) is obtained we solve the system of equation in the form

\[
\tilde{V} = \hat{Z}\tilde{I}
\]

(7.21)

where

\[
\tilde{V} = [\tilde{V}^M \tilde{V}^D]
\]

(7.22)

The metal partition of the solution vector, \( \tilde{I} \), needs to be multiplied by \( j\omega \) afterwards.

### 7.3. Method of calculation the impedance matrix \( \hat{Z} \) and the radiated/scattered fields

#### a. Base integrals and their calculation

Compared to the two particular cases of pure metal and dielectric considered in Chapters V and VI, respectively, Eqs. (7.16) and (7.17) include two new integrals:

\[
\tilde{M}_{MD} = \int \int \tilde{\rho}_i \hat{g}(\tilde{r} - \tilde{r}')d\tilde{r}'d\tilde{s}
\]

(7.23)

\[
\Phi_{MD} = \int \int g(\tilde{r} - \tilde{r}')d\Omega'd\tilde{S}
\]

(7.24)

These integrals (their potential parts) are pre-computed in the script `dielectric.m` in subfolder `2_basis\codes`. The integral (7.24) is not really new and is identical with the
integral (5.20) or (6.23). The singularity extraction and the integral calculation is done following the approach of Chapter V or VI. A “neighboring” sphere of dimensionless radius $R$ is introduced exactly in the same way as for the dielectric (Section 6.9 of Chapter VI). The same default values are used: $R = \sqrt{5}$ and $N = 3, d = 2$ for the surface integrals. These parameters are initialized in the script `dielectric.m` in subfolder `2_basis/codes`.

In integral (7.23), we calculate the inner volume integral first, using the singularity extraction and Eq. (6.28) of Chapter 6. The Gaussian formulas for tetrahedra are identical with those used for the pure dielectric in Chapter VI. The Gaussian formula for triangles (facet of the outer integral) are also identical with those from Chapter VI for dielectric.

b. Solution and filling method

The full impedance matrix is symmetric, but not Hermitian [3]. Therefore, only the upper (or the lower) triangular matrices need to filled out. It is preferred to fill $\hat{Z}^{MD}$ instead of $\hat{Z}^{DM}$ (choose the upper triangular matrix). Only the upper triangular part of $\hat{Z}^{MM}$ and $\hat{Z}^{DD}$ need to be filled accordingly. Then, Eqs. (7.21) are solved using the LAPACK matrix solver `zsysv` with diagonal pivoting for complex symmetric matrices [4].

c. Fields

The scattered fields are calculated separately for metal (Chapter V) and dielectric (Chapter VI), and then are added together. This operation is done in the script `field.m` in subfolder `3_mom/codes`.

7.4. Summary of numerical operations and associated MATLAB/C++ scripts

The full code performs the antenna simulation as described in Section 5.7 of Chapter V. The antenna can include dielectric in any configuration but the feed needs to be specified in the metal as it is done in Chapter V. The full code also performs the eigenfrequency search for a metal-dielectric resonator as described in Chapter VI. However, the `mode.m` (eigenmode field distribution) function is no longer available due to some numerical
difficulties. Instead, `scatterfield.m` may be used to inspect a scattered field at the resonant frequency.
References


